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Road reconstruction and redundancy analysis on the road network: A case study of the Ateneo de Manila University network

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Abstract

We consider studying the redundancy of roads in a given road network during emergency situations. The analysis of link redundancy is interesting especially during disaster mitigation. The roads must be reconstructed as soon as possible to achieve network connectivity. Identifying redundant links would be useful to reduce the cost and guide in prioritization of the road reconstruction during disaster. We presented a model for studying link redundancies using link reconstruction analysis. Our study compares two different techniques in road reconstruction, namely the average impact and the maximum impact road reconstruction techniques. Reconstructing redundant links in a given network reduces the overall distance of the entire network. Our proposed methodology produces similar results, but will vary based on the road network structure.

Keywords: redundancy analysis; road reconstruction; robustness; strongly connected network; disaster mitigation

1. Introduction

Road network is a fundamental part of society. They connect different areas, cities, and people in the best possible way. Different buildings and landmarks can only be visited with the use of certain roads. Studying road networks before, during, and after disaster situation is very important to understand the reliable and robustness of the network. Especially, there is greater need to achieve network connectivity as soon as possible after the network has been destroyed due to the disaster [26]. At the same time, the network has to have the minimum travel cost to reach a certain destination that is solely depended on the behavior of the users [42]

Road networks are becoming more vulnerable to several unforeseen scenarios like disasters, accidents, and other emergency situations, which causes disruptions. This directly affects several aspects of the road network like traffic flow and network connectivity. Furthermore, recovering from such disruptions becomes more difficult overtime due to the changing demands in the network flow, which can directly affect the connectivity of the entire network, as well as the ever-increasing road reconstruction costs.

Studying road network robustness is a relatively new research area. Different research has different definitions for robustness. Several studies been conducted to define several factors in determining robustness, to include identifying critical links, alternate routes, and network connectivity [36,38].

However, only a few studies have been conducted in determining connectivity of the entire network after road reconstruction [8,18,28]. Most road network robustness research only took into account the robustness on an individual link basis without studying how each link affects the entire

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network. These studies also consider other elements like infrastructures, which may aid in connectivity, but is also too complicated to apply because of the usage of images, as well as the inclusion of different road network elements. Although critical links and alternate routes are identified, the network may not necessarily be robust in terms of connectivity due to the limited information it provides.

In this paper, we proposed a new concept of link redundancy analysis to be used in assessing road network robustness. We define a road link as redundant if the existence of the link does not contribute to the connectivity of the road network. The analysis of link redundancy is interesting especially during disaster mitigation when the roads must be reconstructed as soon as possible to achieve network connectivity. Identifying redundant links would be useful to reduce the cost and guide in prioritization of the road reconstruction during disaster.

The scope of this paper does not include the traffic flow of the network. The main focus of the research was performing link redundancy analysis in relation to the entire structure of the road network, without taking into account traffic flow. We also did not include in our research travel costs, in the perspective of a normal commuter.

In Section 2, we defined the terms we used in determining the robustness of a road network. Our actual methodology is found in Section 3, in which each sub-header defined the tools we used to arrive at our analysis. Section 4 discusses an analysis of the network we used to formulate our hypothesis. We applied our methodology in a case study found in Section 5. The scopes and limitations, as well as the conclusions were discussed in Sections 6 and 7, respectively.

2. Definitions and Related Works

To give better understanding of the terms we used throughout the paper, we provide here the definitions and the related works on each part.

2.1 Road Networks

A road network is a weighted directed graph $G = (V, E)$, where V represents a set of vertices (or nodes) and E represents a set of edges (or links). In a road network, the nodes represent intersections among roads, while the links represent the roads themselves. Each edge has a value that represents the distance of the link from a source node i to a destination node j. Certain landmark where the trips are generated is also represented as centroid nodes. A centroid node can either be a source, where the trips are initiated, or a sink, where the trips are terminated, or simply a basin or both source and sink. These centroid nodes are connected to the road network. The direction of each link represents the way the road is traversed. The weight of each link in the road network represents the distance of the network from one node to the next node where the link connects. Road networks are represented based on topology of the network is viewing to a map. The topology is the structure of network in the form of a graph with their respective nodes and links.

2.1.1 Adjacency and distance matrix

An adjacency matrix **A** can be used as a representation of a road network. It is an n by n matrix where n is the number of nodes in the network. The rows of an adjacency matrix represent the nodes of origin of the links, while the columns represent the nodes of destination of the links. A value of 1 is assigned to an element in the adjacency matrix to represent a link from a node to another node. The following equation shows how an adjacency matrix is constructed for road networks:

$$a_{ij} = \begin{cases} 1, \text{if i and j are connected} \\ 0, \text{otherwise} \end{cases} \qquad (1)$$

The values of i and j represent the source and destination nodes in the network, represented by the adjacency matrix **A**. A variation of an adjacency matrix is a distance matrix. However, instead of assigning a value of 1, the actual distance of the link from node i to node j is assigned instead. Otherwise, the value is at 0.

2.1.2. Strongly connected networks

The connectivity of a road network is the minimum number of elements (either nodes or links) removed such that the entire network is disconnected, or divided into two different graphs. A network is connected if a path for every node pair. However, this only applies to undirected networks.

On the other hand, strongly connected networks have a concept similar to connected networks such that every node pair is mutually reachable to one another. In other words, a path exists for every node *i* and *j*, as well as a path from node *j* to *i* for every pair of *j* and *i*. A strong component is a maximal strongly connected sub-network induced on a maximal set of vertices that are mutually reachable [24]. Furthermore, one component of a sub-network is not mutually accessible to another component of a different sub-network [35].

Tarjan [39] proposed an algorithm to find the strongly connected components in a network, which is a variant of a depth-first search. Each sub-network is grouped such that each component is strongly connected to one another. The runtime for the Tarjan's algorithm is at O (|V| + |E|), where m = |V| represents the number of nodes in the network, and q = |E| represents the number of edges in the network. However, this has been greatly reduced to O (|V|) using an ordered binary decision diagram [22].

2.1.3. Shortest path problem

The shortest path problem aims to find a path in a weighted directed graph connecting two node pairs such that the total cost of visiting each path is at the minimum. Different variants of the problem exist, including single-source shortest path problems, in which given a graph $G = (V, E)$, find the shortest path from a source node *s* to every other node *v* in *V*. Another variant of the problem is the single-destination shortest path problem, where a shortest path to a destination node *t* is searched from different node sources *v* in *V* instead of finding a shortest path from a single source node. The single-pair shortest path problem is another problem that is a reduction of the first two aforementioned problems. It focuses on node pairs, such that a shortest path from a source node *s* to a destination node *t* in V is searched [9].

Solving the shortest path problem has been done in the past in the context of network optimization [3,12,14,15,17,23,32] and have been analyzed and evaluated as well in terms of their strengths and weaknesses [9,20,21]. However, most of these algorithms have solved the shortest path algorithm from a perspective coming from one source, one destination, or one node pair only. Another approach is known as the all-pairs shortest path problem [10], which searches for the shortest path from a source node *s* to a destination node *t* for every node pair. This can be solved by running a single-source shortest path algorithm once from each vertex, but can usually be solved faster. Floyd [16] solved the problem using the Boolean matrices theory of Warshall [40]. Another proposition was made by Johnson [30], which solves the same problem, but for sparse networks.

The Floyd-Warshall algorithm [16,40] determines the shortest paths between different node pairs. The algorithm assumes a weighted directed network with at least one directed cycle in its sub-network as an input, where a cycle is a sequence of node traversals such that the start and destination nodes are the same [35]. The algorithm determines the minimum shortest path based on the traversal of the links. The average runtime of the Floyd-Warshall algorithm is O ($|V^3|$), where |V| is the number of nodes in the network. However, Aini and Salehipour [1] proposed a more efficient version of the algorithm. Their version of the Floyd-Warshall algorithm called the Rectangular algorithm was easier to understand and had a better runtime compared to the original Floyd-Warshall shortest paths algorithm.

2.2. Road network robustness during disasters

The robustness of a road network is important, especially during disasters. Robustness is defined as the ability of a network to cope up with exceptional changes like accidents and emergencies [26]. A destroyed link or node can be crucial during these situations. A link is considered destroyed if and only if the origin node is not able to reach the destination node anymore. In a distance matrix, a link is considered destroyed when the matrix element corresponding to the nodes has a value of 0. Several reasons for road link destruction may include natural disasters like floods and typhoons, as well as man-made disasters like terrorist attacks.

2.2.1 Determining robustness criteria

Many criteria have been used to study the robustness of a road network. The research by Dekker and Colbert [11] suggests that node connectivity is the best way to determine network robustness. However, their context was used in network connectivity for communication. Their study did not include the distances of each link, as well as the situation of disconnection during emergency situations.

Sohn [37] presented different criteria for determining the robustness of road networks. His research identified different criteria in determining critical links, which include distance and distance-traffic volume. The latter was more useful in determining the accessibility of each link. However, he also concluded that both criteria would be similar if no alternate route exists in going from one node to another.

Scott, et.al [34] quantified road network robustness index, which yielded different planning solutions compared to the traditional Volume / Capacity (V/C) technique [13]. The premise was that not only should a transportation network meet origin-destination demands, but should also provide connectivity to avoid vulnerability to disruptions. A system-wide perspective was used instead of localized measures, and it was able to provide better indication of the value associated with links in a highway network compared to the V/C ratio. Furthermore, spatial relationships and rerouting possibilities based on network topology were also considered.

2.2.2 Disaster mitigation

Immers, et.al [26] used network structure in order to determine how robust a network is during emergency situations. He proposed a solution that is efficient in handling traffic during normal situations, while providing back-up links used for emergencies only. Their research discovered that several factors affect the robustness of a network, including road redundancy and network structure.

2.3. Road reconstruction

Road reconstruction is an approach used to recover or construct a link that was shut down. Given a network, a road reconstruction algorithm aims to approximately recover links that have been destroyed previously. Roads are reconstructed in order to improve connectivity of a network with respect to topology. A reconstructed road network may have less links compared to the original network, but may improve the shortest minimum distance of the entire network. Having a network with less links but with a longer total shortest minimum path distance is also possible.

Most road network reconstruction algorithms have been used with images generated by geographic information systems (GIS) [6,43]. Other algorithms used genetic algorithms for scheduling during road reconstruction [7]. However, not much study has been conducted in relation to topological road network reconstruction, since it is a design problem.

2.3.1 Road network design problem

Finding the optimal solution to a road network is a design problem in general. The problem determines the optimal configuration of urban network elements, including topology and capacity with respect to a set of criteria [5]. The problem has two forms, namely deterministic and continuous forms.

Most network design problems are focused on finding the optimal cost, capacity, and travel time in a road network [31,33,41]. However, one of the first deterministic solutions to the problem was from Billheimer and Gray [2]. A route selection algorithm was devised while balancing link construction costs and variable user costs. The algorithm was able to eliminate and insert links that converge to local optima. Unique rules are then formulated and must not appear in the global optima. Even though cost was considered, it was effective in reconstructing the optimal roads in the network.

2.3.2. Link Redundancy

A redundant link is a spare link that may or may not affect the overall distance of a road network. Redundancy is the existence of more than one means to accomplish a given function [26]. Furthermore, a lack of redundancy in a network might be catastrophic because of delays, especially during emergency situations, as well as degradation of services.

Adding links to a network can decrease the distance of a network, but may also affect the flow of the network, known as the Braess paradox [4]. A higher cost and travel time may be required when adding a link in a network that is considered a shortcut in the network.

Most studies on link redundancy analysis used alternative routes as an approach to assessing network robustness and redundancy. Jenelius [28] used a rerouting approach in order to study link redundancy in the Swedish road network. In his study, two measures, namely the traffic flow and disruption impact, may affect the network when there are at most two links are disrupted. With traffic flow, most redundant links were located near the largest and longest highway in the network. With the disruption impact, next best alternative routes were considered important. He concluded that depending on the situation, some redundant links might be considered important during extreme situations. The traffic flow criterion can be used for single roads such that it will not affect all other roads, while the impact criterion is useful for roads that could possibly affect other roads when shut down.

Jenelius [27], in his study on network structures, used an equation that computed link redundancy based on the number of links and nodes in a region called the beta index, which was based on the equation from Haggett and Chorley [25]. The index was tested in relation to an alpha index in order to test road network robustness. More alternative routes should in turn create shorter best alternative distances in a road network in terms of the number of links in a road network. By providing alternative routes, the vulnerability of the road network will be reduced. Furthermore, what makes a particular road important can be generalized in larger regions of a road network.

Jenelius and Mattsson [29] conducted a more recent redundant link analysis. A grid-based technique was used and covered covered the Swedish road network based on the extent of link disruption for each cell of the grid. Each cell represented the extent in which the local network would be disrupted. It was concluded that the flow of the link and the availability of alternate routes during disruption determine the magnitude of its impact in a local network. However, disruptions created for more extensive areas tend to have less impact in terms of road network redundancy.

3. Methodology

We divided our network structure analysis into two parts. The first part is the network connectivity, which based links and nodes of the network. The second part is based on distances of each link.

Given a road network, we mapped it using nodes and links. We cleaned the network using Tarjan’s algorithm to solely focus on strongly connected components of the network. We computed the number of links in the strongly connected component in the network, as well as the overall distance of the entire network using the Floyd-Warshall algorithm. Then we destroyed the links in the network less than or equal to a certain threshold value τ. We got some data on how many links were left after destruction, as well as the overall distance of the destroyed network. We also collected information on the number of deleted links in the network, as well as the total distance of the deleted links by getting the sum of the distances. After removal of the links in the network, we calculated the percentage of the network was destroyed after link destruction. We calculated the connectivity percentage of the network to determine how strongly connected the network was. We attempted to reconstruct the network back to a form such that the network was fully connected again using an average or a maximum technique that is based on the impact of each link to the rest of the network. We did this until the network was strongly connected, or in other words, when the connectivity percentage reaches 0. After reconstruction, we calculated the number of redundant links, or links that were not reconstructed in the network, as well as their total distance. We also calculated the number of links in the network after link reconstruction until the network was strongly connected, as well as the overall distance of that network using the Floyd-Warshall algorithm. We then compared distances using a certain distance index to find out if the reconstructed network improved or worsened the overall distance of the entire network.

Given the data from the simulation above, we performed link count and distance analysis in order to assess the relationship between the number of links and the overall distance in the network. We used the case study of the Ateneo network due to its availability of the detail network data in order to test our model. The same methodology can be applied to any road network. We compared the performances of both the average and maximum techniques using statistical analysis. We also used statistical analysis in finding out the relationship between the number of redundant links and the overall distance of the network after link reconstruction.

In this study, we assumed that each link is directed and that each link is subject to a certain distance value. Each link in the network does not have a corresponding travel time, as we strictly focus on the structure of the network alone, without assuming travel time due to the lack of available data in the travel time for each road.

3.1. Network structure

Nodes may be intersections, road corners, or even centroids of another totally different network. Each node has a certain node identification number (node ID). The value of the node ID is arbitrary, and was used to give each node some primitive name representation at the very least.

The roads in the road network are represented as links. The links in the network are directed and connects two nodes, origin and destination. Each link goes only one way, and is dependent on how the road network is structured in the original map. Each road in the network has a distance value specified. The distance is measured in meters (or km), and is an approximation of the distance from one node to another.

When a road network is mapped in the form of nodes and links, the graphical representation of the network is used. A distance matrix, denoted by **D**, is created to represent which nodes are connected to another. An element in **D** has a nonzero value when the two nodes are connected by a link. The origin and destination nodes are represented as the row and column value of **D**, respectively, in which these values come from their respective node IDs. The value of the said element is the distance between the two nodes. A value of 0 is given to an element in a when the two nodes are not connected to one another by a link.

3.2. Data cleaning

Once the distance matrix **D** is formed, the Tarjan's algorithm is executed. The nodes in the network are separated to different groups, known as sub-networks, and each group is the strongly connected components in the network. A node that is strongly connected to another node is grouped together. A node that is weakly connected (or unconnected) to another group is grouped separately from one another. Once the groups have been formed using the algorithm stated, the group with the most number of strongly connected components represents the entire network already. The networks with less number of strongly connected components are removed from the network itself. Finding the largest strongly connected network is important to guarantee that each node is accessible. Each source and sink nodes, as well as the sub-networks associated with them, are removed from the entire network. The distance matrix a is then updated by setting the node ID corresponding to the row and column elements to be equal to 0.

3.3. Links count and distance computation

Using the strongly connected networks, the number of links and distance of the entire network are computed. The number of links in the strongly connected network is represented by the variable n_a. A binary matrix, denoted by **B**, is computed in relation to the distance matrix **D**. The binary matrix **B** is an m by m matrix, where m is the total number of nodes in the network after data cleaning. Each element in **B** corresponds to an element in **D**. For each element in **D**, if the distance value is non-zero, the value of the corresponding element in **B** is 1. Otherwise, the value of the corresponding element in **B** is 0. The variable n_a is the sum of the elements in **B**, which is given by the following formula:

$$n_a = \sum_{i=1}^{m} \sum_{j=1}^{m} \mathbf{B}_{i,j} \quad (2)$$

where i and j represent the row and column addresses of the network, respectively, corresponding to the node IDs of the network, and the variable n represents the number of nodes in the network.

The distance of the entire network is represented by the variable d_a. The Floyd-Warshall algorithm is used to calculate the value of d_a. Given the strongly connected network, the distances are calculated from one node to another node by adding the total minimum link distances going to the target node. The results are stored in a separate matrix, denoted by **S**. The matrix **S** is an m by m matrix, where m is the total number of nodes in the network, such that each element has a value corresponding to the minimum distance of going from one node to another node in the network. The rows represent the starting node, while the columns represent possible destination nodes in the strongly connected network. A value of 0 is assigned to the element in the matrix if there is no possible way of reaching the destination node from the source node. Once the distances are computed by the said algorithm, the sum of the matrix elements is computed and stored in d_a using the following formula:

$$d_a = \sum_{i=1}^{m} \sum_{j=1}^{m} \mathbf{S}_{i,j} \quad (3)$$

where i and j represent the row and column address of the network, respectively, which correspond to the node IDs of the network, and m represents the total number of nodes in the strongly connected network. The variable $\mathbf{S}_{i,j}$ is the minimum distance value of the link from node i to node j calculated by the Floyd-Warshall algorithm. The value of d_a is always constant for one particular network, and will only change when a different network is presented.

3.4. Link Destruction

Once the values of n_a and d_a are computed, the network is now subject to a simulated road destruction within a certain threshold value, denoted by τ. The variable τ is a random number that represents the threshold that determines which link is to be removed. Increasing the value of τ means that more links will be removed, while a decrease in the value of τ means that fewer links in the network will be removed. When the value of τ is reduced to less than the minimum link distance value in the network, no links will be destroyed. However, when the value of τ is greater than the maximum link distance value in the network, then all nodes are subject to destruction. Each link in the network is compared to the value of τ. If a link is less than or equal to τ, that link is removed from the network. On the other hand, when a link is greater than τ, it remains in the network and is not removed.

Before the link is removed from the network, a copy of the link is stored in a separated distance matrix, denoted by **C**. The separated distance matrix **C** is an m by m matrix. When a link is designated for removal, a copy is stored in **C** at the address where the link is located. The address assignments of **D** and **C** are identical, in which each row represents the source nodes, and each column represents the sink nodes. When a copy of the link to be removed is stored in **C**, the link is removed from the network. The corresponding distance matrix **D** is updated by assigning the distance value of the corresponding element to be 0 to denote that the link is removed from the network. This procedure is called the link destruction process.

3.5. Link Count and Distance Computation After Link Destruction

After destroying links in the network, the number of links, denoted by n_0, is computed. The variable n_0 is the sum of the total number of links after some links in the original network have been removed. The value of n_0 is the total number of links remaining in the network. The approach used in Section 3.3 using Equation 2 was the similar approach used in computing n_0. A higher value of n_0 meant that there were more links that were not deleted. This implies that there were less links removed due to the fact that the random value of the threshold τ was very low. When the value of τ was higher, the value of n_0 decreases because the higher the value of τ is, the more links that were destroyed in the network.

The distance of the network after link removal is represented by the variable d_0. The similar approach of Equation 3 found in Section 3.3 was used for calculating the value of the total distance of minimum paths, which is d_0. When the value of d_0 was higher, the links with the shortest paths were removed during link destruction. When the value of d_0 as lower, the links in the network were less connected with one another because of the lower value of n_0. This implies that the threshold value τ was higher as well that deletion of links that were crucial to the strength of the connectivity of the entire network.

3.5.1. Percentage Calculation

We calculated the percentage connectivity, denoted by p, of the entire network using the Floyd-Warshall algorithm. The percent completeness of the network was computed using the similar approach used in getting the distance in Section 3.2. However, the difference is that instead of adding the total minimum path distances of the network, the total number of nonzero elements in the matrix **S** was computed. Given a distance matrix **S**, a corresponding binary matrix $\mathbf{B}_p$ was calculated. The binary matrix $\mathbf{B}_p$ is an m by m matrix, where n corresponds to the number of nodes in the network. The binary matrix $\mathbf{B}_p$ may either have a value of 1 or 0. For each element in the matrix **S**, when the value of the element in **S** is a nonzero value, then the corresponding element in $\mathbf{B}_p$ is assigned a value of 1. Otherwise, if the element in **S** has a value equal to 0, then the corresponding matrix in $\mathbf{B}_p$ is assigned a value equal to 0.

When every element in the matrix **S** has been checked, the binary matrix **B** is complete. Then the sum of the elements in the binary matrix **B** is computed. The sum of the elements in **B** is then divided by the square of the number of elements in the network. That result was then subtracted from 1 in order to evaluate the value of p. The following formula was used in calculating the value of p:

$$p=1-\frac{\sum_{i=1}^{m}\sum_{j=1}^{m}\mathbf{B}_{i,j}}{m^2} \qquad (4)$$

where i and j represent the node IDs of the rows and columns, respectively, which correspond to the address of the binary matrix **B**, and n represents the number of nodes in the network. A higher value of p means that the nodes in the network are less connected to one another. This implies that the connectivity of the network is not good. However, a lower value of p means that the nodes in the network are more connected and more accessible to one another. This means that from a given node, going to another node is much more possible. The optimal value of p is at 0, which means that the nodes in the network are accessible from one node to another. However, when p is 1, it means that the nodes are totally separated from one another, since there are no links connecting networks to one another. This is caused by a τ that is higher than the maximum distance value of the links in the original network.

3.6. Counting the Number of Links Removed and Their Total Distance

The number of links removed during link destruction, denoted by n_d, is also computed using the distance matrix **C**, which was calculated using the link removal approach used in the preceding statements. The value of n_d is the total number of links removed during link destruction. When the value of n_d is 0, it means that no link was destroyed given the threshold value τ. A higher value of n_d means that more links were removed during the link destruction. The similar approach in Section 3.3 was also used in computing the value of n_d, except that the distance matrix **C** was used instead of the distance matrix **D**, since this represented the distance matrix of the entire network.

The total distance of the links removed during link destruction was also computed, denoted by the variable d_d. However, instead of using the Floyd-Warshall algorithm, as explained in Section 3.3, the sum of the distances in the distance matrix **C** was computed instead because the deleted links in **C** may not necessarily intersect with common nodes as source or sink nodes of another link. In this case, links are considered to be unrelated to another. The formula used for calculating the total distance of the links deleted was defined in in this equation:

$$d_d= \sum_{i=1}^{m}\sum_{j=1}^{m}\mathbf{C}_{i,j} \qquad (5)$$

where i and j represent the node IDs of the rows and columns, respectively, which correspond to the address of the distance matrix **C**, and m represents the number of nodes in the network. The variable $\mathbf{C}_{i,j}$ represents the distance value of the link connecting nodes i and j. When the value of d_d is at 0, no links were disconnected from the network under the random threshold value τ. However, a higher value of d_d meant that the total distances of the links were destroyed were high as well, perhaps due to the fact that the threshold value τ is higher as well. In other words, the threshold value τ and the value of d_d is directly proportional to one another because the value of d_d is the sum of all the distances of the links removed during link destruction, which are less than or equal to τ.as explained in the previous section.

3.7. Link Reconstruction

Once the links in the network have been destroyed, the next step is to reconstruct the links. This was done in order to find out what the redundant links are in the network, as well as assess the completeness of the network when those links are not present in the network. The goal of the link reconstruction is to reconstruct links until the network is once again strongly connected. Link reconstruction will continue until the network formed is strongly connected. The value of p, which was the connectivity percentage of the entire network computed in Equation 4, is reduced to 0. When the value of p is equal to 0, it means that the network is strongly connected upon link reconstruction.

3.7.1. Assumption for Reconstruction of Deleted Link

Given the distance matrices **D** and **C**, which represent the network after link destruction and links that are to be reconstructed, respectively, we want to reconnect the entire network using the list of deleted networks until the network is strongly connected. Each nonzero link element represented in **C** is put to the corresponding address of a to assume that the link exists. This is called assuming the link for reconstruction because after determining the impact of the link to the entire network, denoted by Δ, the link will then be removed from the network. When the link is connected, the Floyd-Warshall algorithm was used to determine the connectivity of the network when link was present. Given a separate matrix **S**, which is an m by m matrix, where n is the number of nodes in the network, an element in the matrix **S** has a value equal the total minimum distance to reach from the source node to the destination node. When the source node does not have any way to reach the destination node, the value of the element in **S** is equal to 0.

When the matrix **S** is already complete, a binary matrix, denoted by **B**, was computed. The binary matrix **B** is an m by m matrix. Only the values of 0 and 1 are used for the binary matrix **B**. A value of 1 is denoted for an element in **B** when the corresponding element in the matrix **S** is non-zero. Otherwise, when the value of the corresponding element in **S** is zero, the value of the element is 0.

When each element of the matrix **S** has been visited, the binary matrix **B** is now completed. The sum of the elements in the binary matrix **B** was then computed and was divided to the square of the number of nodes in the network. The result was then subtracted from 1, which results into the percentage value p_d. The following formula was used to compute the value of p_d:

$$p_d = 1 - \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} B_{i,j}}{m^2} \quad (6)$$

where i and j represent the node IDs of the row and column, respectively, which correspond to the address of an element in the binary matrix **B**, and n represents the number of nodes in the network. When the value of p_d is lower, it means that the impact the link makes when reconstructed to the network is lesser. When the p_d value is higher, it means that the impact the link makes when reconstructed to the network is greater, thus improving the connectivity of the network. Each link has a different p_d value, depending on how much the network improved when the link was added to the network.

3.7.2. Delta Calculation

Once the value of p_d is computed, the next step is to find the impact of the link to the entire network and how it affects the entire network. Given the values of p_d and p, the goal is to calculate their difference, denoted by Δ. The Δ value represents the impact that the link has over the entire network. The corresponding p_d value determines the importance of the link to the connectivity of the entire network. The formula for the Δ value was computed using the following equation:

$$\Delta = p_d - p \quad (7)$$

where p_d is the percentage of the network when the corresponding link was added to the network, and p is the percentage of the network after link destruction. A higher Δ value means that the value of p_d greatly the connectivity of the network using the corresponding link to the given pd. However, when the value of Δ is lower, the pd is lower, which means that value of p_d improved the connectivity of the network to a much lesser extent when the corresponding link was connected.

The values of Δ are different depending on the link currently assumed for reconstruction. When the delta value is computed, the corresponding link in the network will be deleted once again by assigning the value of 0 to the corresponding address of the element in the distance matrix a. The delta valued is then stored in an array, denoted by Δ_{arr}. The process of assuming each link for reconstruction will be done until all links have been assumed for reconstruction with a corresponding Δ value stored in the array Δ_{arr}.

3.7.3. Average Delta Technique (ADT)

After assuming each link in the distance matrix **C** for reconstruction, the next step is the actual reconstruction of the links in the network. Given the array of the delta values Δ_{arr}, the average of the array value, denoted by Δ_{avg} was computed using the following formula:

$$\Delta_{avg} = \frac{\sum_{i=1}^{m} \Delta_{arr_i}}{k} \quad (8)$$

where i is the corresponding address of an element in Δ_{arr}, and k represents the number of elements in Δ_{arr}. Computing the average is the first part of the ADT.

The next part of the ADT is the link reconstruction. Using the distance matrices **D** and **C**, which represent the network after link destruction and the list of links to be reconstructed in the network, respectively, the specific links are reconstructed. Every non-zero element in **C** has a corresponding Δ value, which was computed in the previous section. If the delta value of the element in **C** is greater than or equal to the average, the link is immediately reconstructed to the network represented by the distance matrix **D**. The element in **C** is then assigned to the corresponding element of the distance matrix a, with a value equal to the distance between the nodes it represents, which are the row and column of the corresponding element, respectively. The same element in the distance matrix **C** is then assigned a value of 0, indicating the deletion of the link from the list of the networks to be reconstructed.

After reconstructing the entire network, Equation 4 is once again used in order to re-compute the percentage of the network after one instance of link reconstruction. The similar technique and same assumptions in Section 2.5.1 was used to recalculate the value of the percentage.

3.7.4. Maximum Delta Technique (MDT)

After assuming each link in the distance matrix **C** for reconstruction, the actual reconstruction was conducted. Given an array of delta values, denoted by Δ_{arr}, the maximum Δ value in the Δ_{arr} was computed, denoted by Δ_{max}. Every non-zero element in **C** has a corresponding Δ value computed in Section 2.7.2. If the delta value in the Δ_{arr} is equal to Δ_{max}, the link corresponding to the Δ value in the Δ_{arr} is reconstructed to the distance matrix **D**. The element in **C** that is to be reconstructed is then assigned to the corresponding address of the distance matrix a. At the same time, the element in **C** is assigned a value 0, indicating deletion of the link from the list of networks to be reconstructed.

3.8. Links Count and Distance After Reconstruction

After link reconstruction is finished, the network is now strongly connected. When the value of p reaches a value of 0, it means that after link reconstruction, the network is now strongly connected. Computing the number of links after reconstruction, denoted by n_1, was executed using the similar approach in counting the different number of links used in Section 3.3 using Equation 2. A higher value of n_1 meant that roads were reconstructed in the network. When the value of n_1 is similar to the value of n_0, which is the number of links after reconstruction, as stated in Section 3.5, no links were reconstructed in the network. This is caused by a very low threshold value, which meant that fewer links were deleted. The value of n_1 will always be greater than the value of n_0, but may also have a value equal to n_a, which means that all the links were reconstructed in the network.

The distance of the network after reconstruction is denoted by the variable d_1. The network is guaranteed to be a strongly connected network, since the value of p is now 0, on the link reconstruction approach done in Section 3.7. The computation for the value of d_1 used the a similar approach to Section 3.4, while using Equation 3 at the same time to calculate the overall sum of the minimum distances calculated by the Floyd-Warshall algorithm.

The value of d_1 is always greater than the value of d_0 due to the added number of links included in the distance matrix a after link reconstruction. When the value of d_1 is greater than the value of d_a, the network after reconstructing the links destroyed was more inefficient because of the lack of paths that would otherwise be selected by the Floyd-Warshall algorithm as a candidate for minimum path selection.

On the other hand, when the value of d_1 is smaller than the value of d_a, the network after road reconstruction was more efficient in defining the critical links. Aside from the network being strongly connected after link reconstruction, the network was also made more efficient in relation to the overall sum of the minimum distance of the network. This means that unnecessary links were removed and were considered redundant because they do not help in the improvement of the connectivity of the network.

The value of d_1 will only be equal to the value of d_a if the link reconstruction resulted into the original network before links were removed. This case is less likely to happen because as stated earlier, different road networks exist for different areas.

3.9. Links Count and Distance of Redundant Links

When the links are reconstructed to the entire network, as shown in Section 3.8, a possible number of links may remain in the distance matrix **C**, which is the matrix representing the destroyed links. The variable q denotes the number of links that were not reconstructed on the network. The similar approach to counting the number of links in Section 3.3 using Equation 2 was used to get the value of q. The value of q will only be equal to 0 if and only if all the destroyed links have been reconstructed to the network. This happens when all elements in the distance matrix **C** are equal to 0.

A redundant link is a link that does not affect the connectivity of the network when added. Even without these links, the network may still have strongly connected. The links remaining in the distance matrix **C** were the nonzero distance values of the matrix, and were considered redundant. The variable d represents the total distance of these redundant links. Since these links may not necessarily intersect in a similar node, the Floyd-Warshall algorithm was not used. The same approach that Section 3.5 was used in calculating the value of d as well. Equation 5 was used in computing for the total distance of the deleted links in the network.

3.10. Distance Index

The distance index is a measure used in determining how different the network was to the original network structure after link reconstruction. This is computed after network reconstruction. The variable DI denoted the distance index. The following formula was used in order to compute the value of DI:

$$DI = 1 + \frac{d_a - d_1}{d_a} \quad (9)$$

where d_a represents the overall distance value of the entire network, which was computed using Equation 2. The value of d_1 represents the distance value of the entire network after link reconstruction using the similar Equation 2. A DI value of 1 meant that the reconstructed network was identical to the original network. In other words, when the DI value is 1, no redundant links were present in the network, and that each link was important in the network. When the value of DI was greater than 1, the original network had a worse overall distance value compared to the network after destroyed links were reconstructed. This meant that the redundant links in the network were detected in the network, thus decreasing the importance of these links to the connectivity of the network. However, when the value of DI was lower than 0, the original network had a better overall distance value compared to the network after destroyed links were reconstructed. This meant that the network after reconstruction was more costly compared to the entire network, thus increasing the importance of redundant networks. A summary of this is shown in the following conditional equation:

$$\mathrm{DI} = \begin{cases} <1;\ \text{if } d_1 > d_a \\ =1; \text{if } d_1 = d_a \\ >1; \text{if } d_1 < d_a \end{cases} \qquad (10)$$

4. Network Analysis

This section is the analysis of the methodology performed in Section 3, as well as some equations used in order to assess road network redundancies.

4.1. Node Count Analysis

To analyze the data, we classified data into two groups for analysis in the relationship between the distance and the number of redundant links in the network. One property in determining the relationship in between the number of links is shown in the following equation:

$$n_0 + n_d = n_1 + n = n_a \qquad (11)$$

When the number of destroyed links, denoted by n_d, and the number of links remaining in the network after link destruction, denoted by n_0 is added together, the result is the total number of links in the original strongly connected network, denoted by n_a. The similar case is also true when the number of redundant links, denoted by n, is added to the number of links in the network after link reconstruction, denoted by n_1. This means that the links currently in the network and links deleted from the network will always complement with one another. Figure 1 represents the visual representation of the computation of each value in the Equation 11.

However, we present a new metric, denoted by φ, to represent the ratio between the number of redundant links and the number of destroyed links in the network after link removal. This was done in order to show how much of the redundant links have been detected in relation to the number of links removed in the network. In other words, φ determines the percentage of links from the total number of destroyed links that are considered redundant. A higher value of φ means that more redundant links are present in the list of deleted links in the network. The following formula is used for computing the value of φ:

$$\varphi = \frac{n}{n_d} \qquad (12)$$

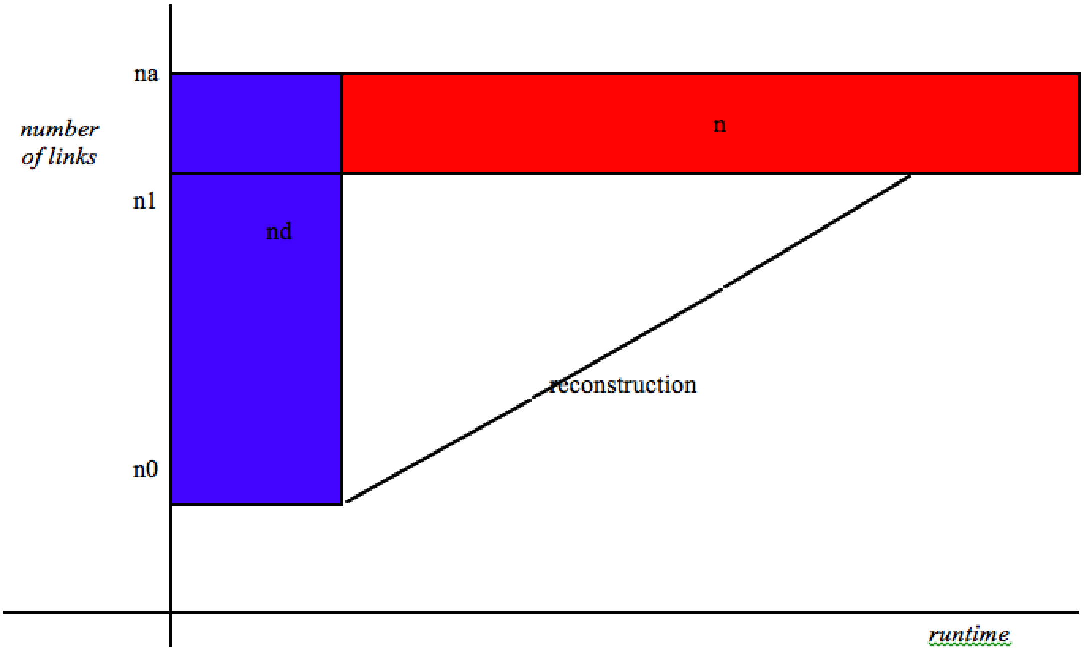

Fig.1. Representation of computation of the number of nodes in the network

4.2. Distance Analysis

On the other hand, the link distances of the data behave rather differently compared to the ones shown in the node analysis in Section 3.1. The overall distance, denoted by d_a, is computed as the sum of the overall distance of the network after link destruction, denoted by d_0, and a certain complementary value, denoted as Δ_d. The value of Δ_d represents the difference of the distances between the original overall distance and the overall distance after link destruction. The following formula is computed in order to get the value:

$$\Delta_d = d_a - d_0 \quad (13)$$

The value of Δ_d may either be positive or negative. A positive value of Δ_d means that the original distance is higher compared to the distance of the entire network after link destruction. This is the common case since a greater τ value, which represents the threshold to link destruction explained in Section 3.4, means a greater distance taken away from the entire network. Increasing the value of the Δ_d means that the distance value d_0 is far away from the original distance d_a. We have established in Section 3.3 that the value of d_a never changes for a particular network. In other words, a higher Δ_d implies that the topology of the network after link destruction gets further from the original network, at least in terms of link distance. It is also possible to have a negative value for the Δ_d, which means that the distance of the original link was better compared to the distance of the link after link destruction. This means that some critical links were removed from the network. As a result, the Floyd-Warshall algorithm detected the next minimum link in one link during the calculation the overall minimum distance.

On the other hand, to determine the closeness of the overall distance after link reconstruction to the overall distance of the network, the variable Δ is used. Note that the Δ in this section is totally different from the Δ computed in Equation 7 from Section 3.7. The following formula was used to compute the value of Δ:

$$\Delta = d_a - d_1 \quad (14)$$

The value of Δ may be either positive or negative. A positive Δ means that the value of d_1 is lower than the value of d_a, which means that the overall distance of the network decreased after link reconstruction. Increasing the value of Δ means that after link reconstruction implies that the topology of the network after link reconstruction gets farther from the original network in relation to link distances. When the value of Δ is negative, the value of d_a is lower than the value of d_1, which means that the overall distance of the network increased after link reconstruction. And as stated in Section 3.8, a higher value of d_1 means that important paths that were classified as redundant were removed from the network, thus affecting the efficiency of the network in relation to the total minimum distance computed from the Floyd-Warshall algorithm. The computation of all the aforementioned variables is further visualized in Figure 2.

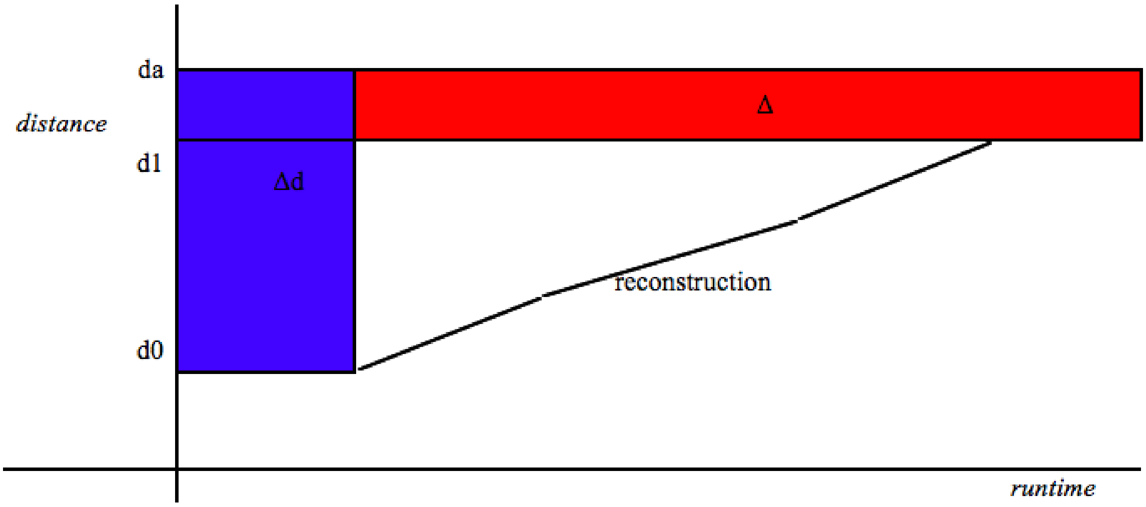

Fig.2. Representation of the computation of distances and their respective differences

We also present another metric, denoted by ξ. This represents the ratio between the difference of the distance after and before reconstruction, denoted by Δ and Δ_d, respectively. This was calculated in order to assess how much the difference of the distances has improved after the links were reconstructed in the network compared to before the links were destroyed. The following formula is used for computing the value of ξ:

$$\xi = \frac{\Delta}{\Delta_d} \quad (15)$$

5. Case Study: The Ateneo de Manila University Network

To illustrate the concepts explained in the previous section, we used the Ateneo de Manila University Loyola Schools road network in order to test our proposed model due to its availability of the detail measurements. The road network, as shown in Figure 3, had 76 nodes and 136 links. The network model used was only effective last 2012 [19]. We used the base scenario of the road network for our case study. There were 40 dummy links present in the network to connect to the centroids. The role of these dummy links was to represent a larger scale network totally unrelated to the Ateneo road network.

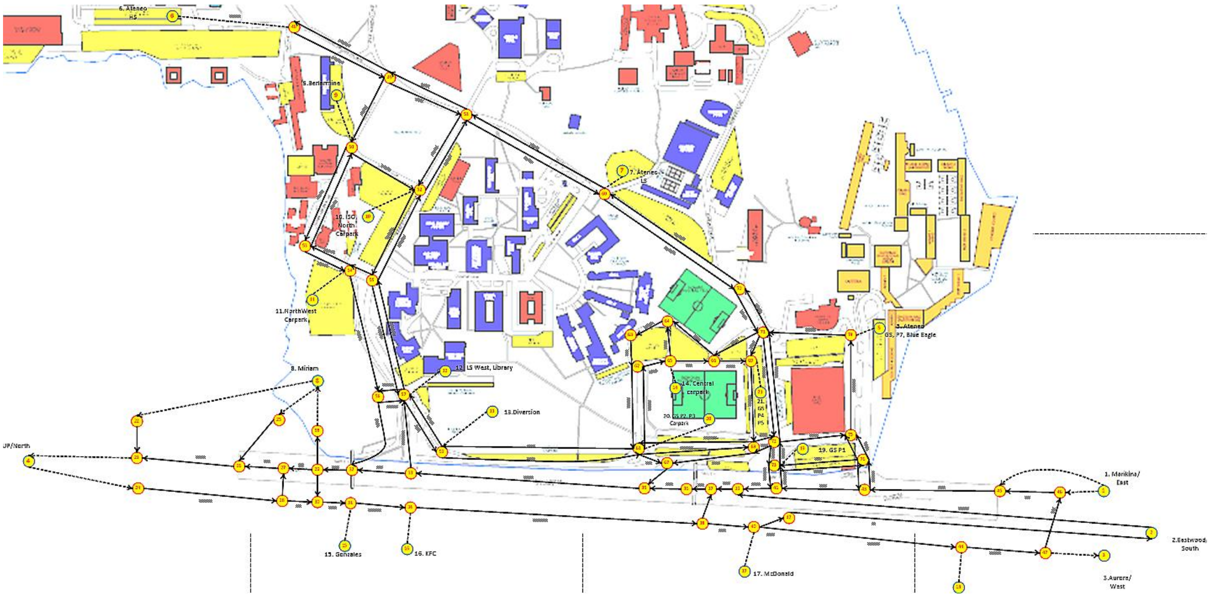

Fig.3. Ateneo Road Network Map [19]

Excluding the dummy links, the total distance of the Ateneo network, when added as individual links, 8843.2 meters. The overall distance, using the Floyd-Warshall algorithm, for the Ateneo network was 4,860,698 meters. Our analysis set a fixed τ-value ranging from 10 to 400 with increments of 10. Both the average and maximum approaches of road reconstruction were used and compared to one another, as well as studying the relationship of the number of links to the overall distance of the network.

5.1. Average Technique Analysis

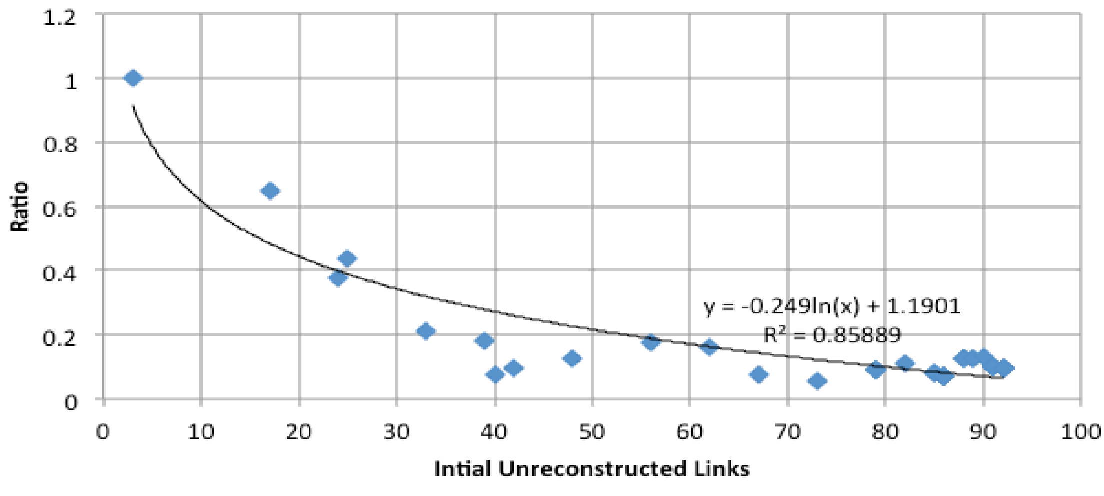

Fig.4. Relationship of the Number of Links to be Reconstructed and the Ratio with the Number of Redundant Links for the Average Technique

The average technique yielded an average runtime of 35.76 seconds. Figure 4 shows the relationship between the numbers of links to be reconstructed to its ratio with the number of redundant links for each tau-value from 10 to 400 with increments of 10. We discovered that a logarithmic pattern was the best fit in this graph, since it yielded an R-square value of 0.86. The logarithmic behavior is supposed to happen because the higher the number of links that should be reconstructed, the less the ratio gets.

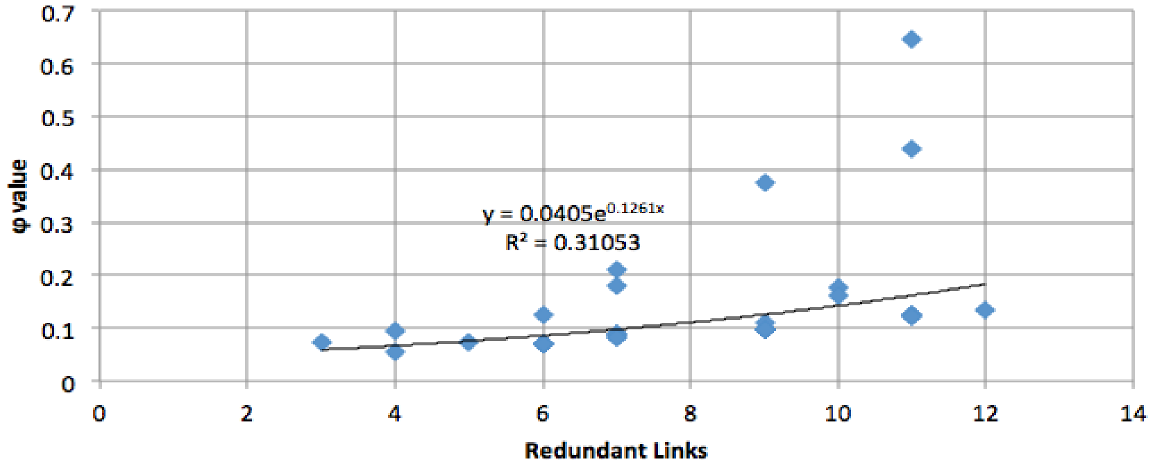

Fig.5. Relationship of the Number of Redundant Links and its Ratio to the Initial Links to be Reconstructed for the Average Technique

However, even though the number of unreconstructed links was higher and more logarithmic, as shown in Figure 4, the behavior of the redundant links and ratio shown in Figure 5 is exponential. The plot shown here did not include the tau-value of 10. This τ-value yielded 3 links to be reconstructed and all 3 links were redundant. It was not included in Figure 4 because it greatly affected the variance. As seen in Figure 5, without the τ-value of 10, the R-square value is at 0.31. The model behaved in an exponential pattern, as it is the inverse of the logarithmic curve. The R-square value in Figure 5 was low because although the fit was wrong, the trend was not good because of the high variance of the datasets. What happens is that as the redundancy increases, the variance gets larger, as seen in the plot. When the value of the redundant links is less than 8, the points were relatively near to the exponential curve. But as the number of the redundant links increases, the variance starts to be greater, thus resulting into a low R-square value.

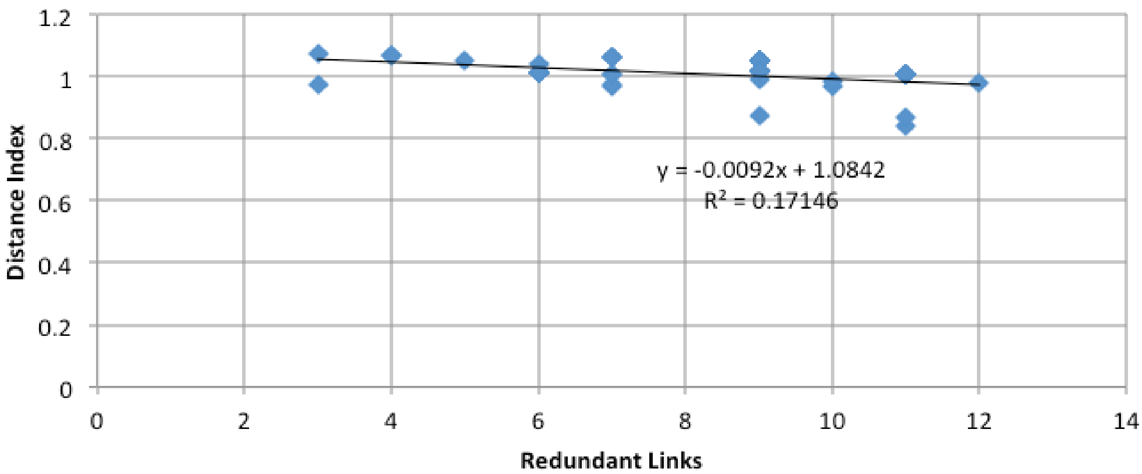

Fig.6. Relationship of the Number of Redundant Links to the Distance Index for the Average Technique

The relationship between the distance index and redundant links for the average approach, as shown in Figure 6, is almost a flat line. The value of the R-square was very low, even though the points were visually near to the line of best fit. Note also that some points in Figure 6 went beyond the distance index value of 1, which means that for some value of the threshold τ, the distance of the network improved even without putting the redundant links in the network. However, the downward trend can be seen in Figure 6. This means that as the number of redundant links increase, the value of the distance index decreases as a result of increasing the overall distance of the network. At the same time, the trend went beyond 1 when the number of redundant links was less than 8, which implies that in the Ateneo network, when the average technique is used, the redundant link less than 8 yielded a better overall distance in the network.

5.2. Maximum Technique Analysis

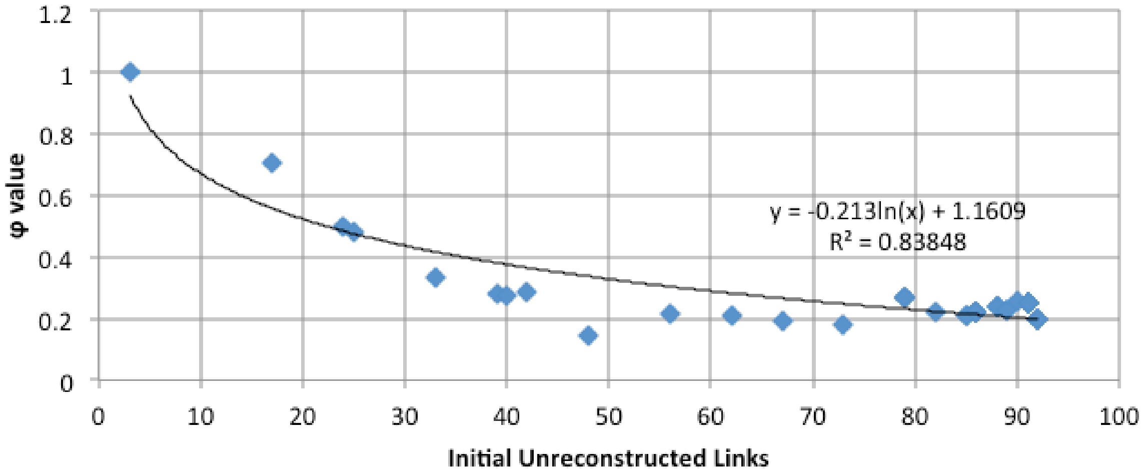

Fig.7. Relationship of Number of Unreconstructed Links to Ratio to Number of Redundant Links for the Maximum Technique

The maximum technique had an average runtime value of 313.59 seconds. As shown in Figure 8, the maximum technique yielded a logarithmic behavior between the number of links to be reconstructed and its ratio to the number of redundant links, as shown in Figure 7. This is similar to the behavior of the average technique shown in Figure 4. However, the R-square value is slightly less than the average approach, with an r-square value of 0.84. In other words, both graphs in Figures 4 and 7 clearly explain that as the number of unreconstructed links increases, the ratio to the number of redundant links diminishes. This is supposed to happen because a higher tau-value means more links need to be reconstructed, and since the ratio is the number of redundant links over the number of links to be reconstructed, the ratio goes down.

However, the number of redundant links varies as well in the maximum technique, like the one in the average technique shown in Figure 4. The r-square value was very low due to its large variance. In other words, the number of redundant links is highly variable, depending on the completeness of the paths, as well as the paths reconstructed.

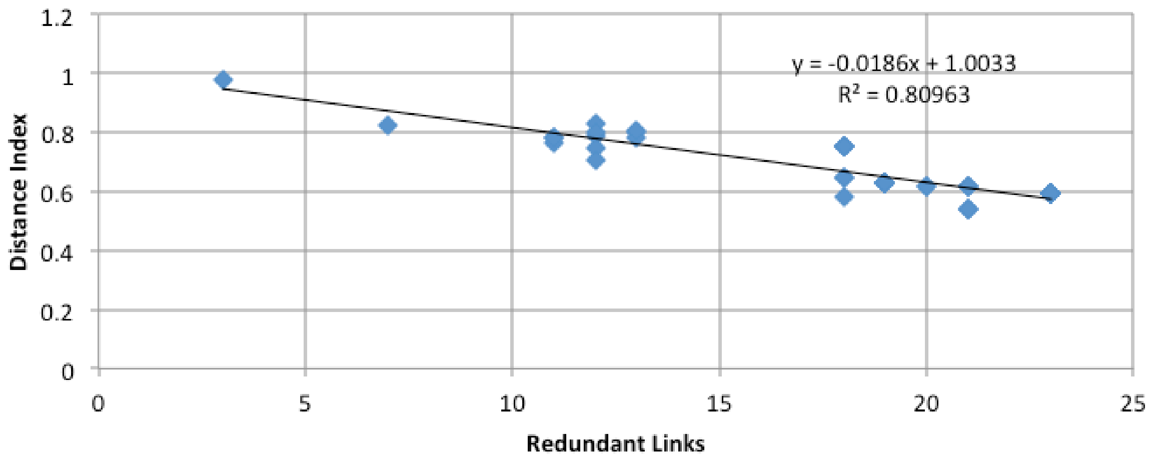

Fig.8. Relationship of the Number of Redundant Links to Distance Index for the Maximum Technique

A downward trend was shown in the relationship between the distance index and the number of redundant links for the maximum technique, as shown in Figure 8. This was fairly consistent in relation to the one shown in Figure 6, even though the slope was lower compared to the maximum approach. In other words, as the number of redundant links increase, the distance index decreases, which makes sense because the more redundant links are removed from the network, the less identical it is from the original network. This implies a worse distance for the reconstructed link. By adding this redundant link to the network, the connectivity and the overall distance of the network improves.

4.3. Relationship Between Number of Links and Overall Distance

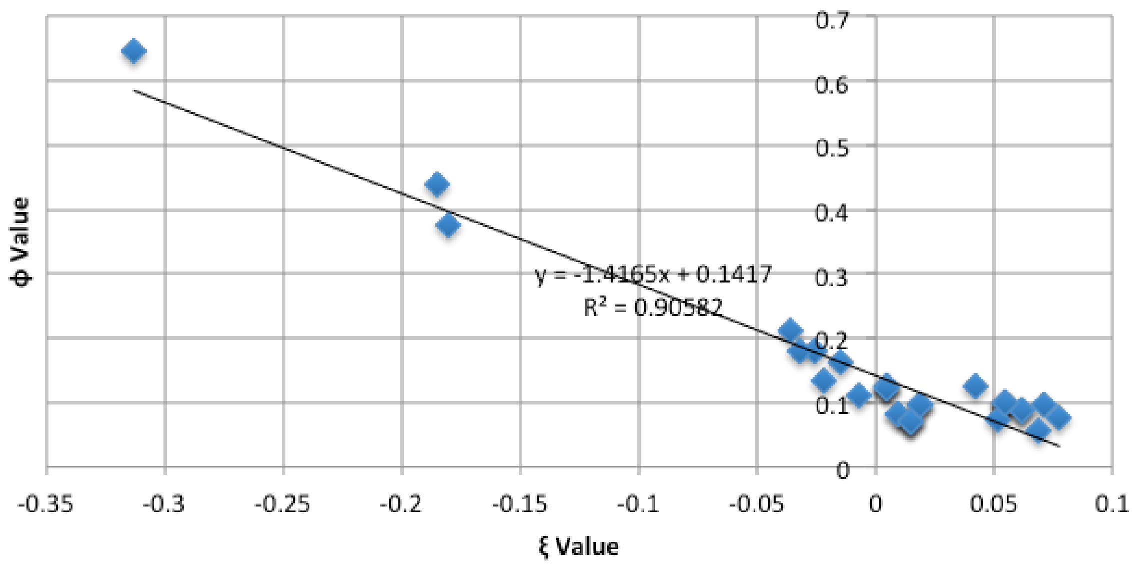

Fig.9. Relationship between the values of ξ and φ, representing the ratios of the distances and the number of links, respectively

Figure 9 shows the relationship between the two ratios stated in Section 3, as calculated by Equations 12 and 15. A strong relationship between the distance of the links and the number of links exists, as explained by the R-square value of 0.91. As a result, increasing the number of redundant links results into a greater value of the distance after road reconstruction. Therefore, Figure 9 also supports the statement that including redundant links in the network will improve the distance of the entire network, as shown by the trend of the graph.

6. Conclusions

Based on the proposed concept and the analysis of the case study above, we concluded that reconstructing a destroyed link makes the path of the entire network shorter, provided that the link reconstructed is a redundant link. Adding redundant links does not necessarily add to the connectivity but it improves to make the travel distance shorter, assuming that no other roads are closed [27]. We concluded this because of the downward trend between the number of redundant links and the distance index of each link. Even though the maximum technique ran slower than the average technique, we also found out that both approaches had a logarithmic behavior in comparing the value of ϕ and the links destroyed in the network. This is especially true since the higher the value of τ, the more links that would be destroyed during link destruction.

In comparing the average and maximum techniques for our road reconstruction techniques, we conclude that although the average technique may be faster, the results it produced were more variable compared to the data produced by the maximum approach. However, both techniques confirmed a downward trend of the number of redundant links to the distance index.

In summary, we have formulated a new model for studying road redundancies and how it affects road connectivity. We would like to use this concept as our criteria for road robustness. However, we were not able to conduct a more detailed comparative analysis of which technique would be better in road reconstruction. Our study did not provide alternate routes because we tested it from the topological perspective, which implies that we tested our model as a whole and not as parts, unlike other studies done.

A possible future work would be to improve the road reconstruction technique we used, such that instead of using the percentage of the network upon assumption for road reconstruction, what we plan to do is to use the overall distance of the network to assess how much the distance was improved during road reconstruction. We would also be open to further analyses of our model, like including traffic flow as well in an attempt to create a more robust redundancy analysis model for assessing road redundancy. Another possible study would be to focus on each region in our model in order to assess its local impact to the entire network, which was similar to the grid-based technique proposed by Jenelius and Mattsson [29]. Our proposed model may also be recreated in different networks in order to determine if the model we established is a useful model the next time road network robustness analysis is conducted.

Our research only focused on network structure and not the network usage. The research did not also take into account the disaster type during disaster mitigation of the network. Our study assumed that a generic disaster was present in the road network. Further studies may be conducted in the future implementing these additional factors in the redundancy analysis, specifically on the effects of the flow in the network when there are disruptions such as these. Furthermore, we would also include in our future work stochastic techniques that may affect how road networks deteriorate not just because of disasters, but also because of constant wear-and-tear to assess the performance of a given road network.

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