

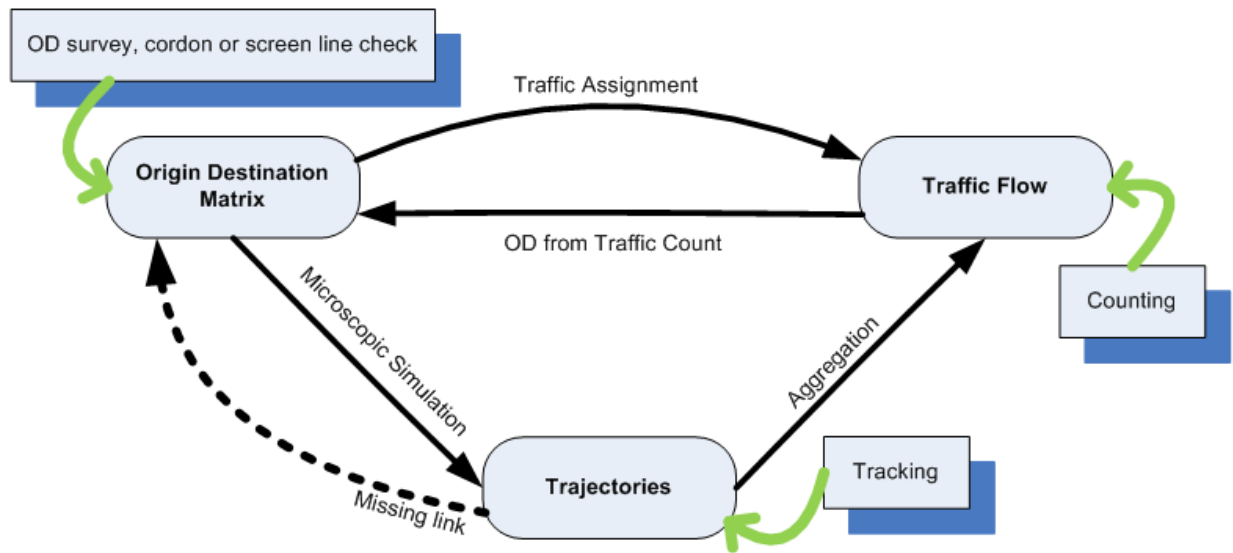
A Theoretical Foundation for the Relationship between Generalized Origin Destination and Flow Matrix based on Ordinal Graph Trajectories

Abstract

*r z d r r st d dzst r dst dd ! ftd d z dch ft clrst zst z chz st zst d st
z rdst ch z ftz stz d st dr stzrstst stz zrr ft d st dst ch stzst
zst st ch d zst ! d rd ch z stz d st z dst ij ftz zr stz ch ch d st
std ftd d z dch zst z chstd zst ! st stdz st zst dz chr rststst st zst dr zr
zclhst z st st Tr ft dz z ftd z ijd d zst r zst rdst std dzst r dst dd dst ij
st zst st r ! ftd d z dch ! z st zst d st z chclst d d z ch dst ijrst st d
st r chst d zst z chzchz d zst z dclst dch*

Keywords

1. Introduction



$d_{st} \quad ij \quad st \quad z_{st}$

$d_{st} \quad ij \quad r_{st} \quad st \quad d$

1.1. Definitions

$$s\mathfrak{t}z\ d\ s\mathfrak{t}$$

$$t_1\quad t_2\qquad t_1\quad t_2$$

$$\begin{matrix} n \\ X_n(t)\quad x_n(t)\quad y_n(t)\quad z_n(t)^T \end{matrix}$$

$$s\mathfrak{t}$$

$$c\mathfrak{h}\ z\ s\mathfrak{t}z\ d\ s\mathfrak{t}$$

$$\begin{matrix} s\mathfrak{t}\ s\mathfrak{t}\quad s\mathfrak{t} \\ \dot{y} \end{matrix}$$

$$\ddot{y}\quad \ddot{y}\quad \ddot{y}$$

$$\begin{matrix} c\mathfrak{h}\ z\ f\mathfrak{t}\ z\quad s\mathfrak{t}z\ d\ s\mathfrak{t} \\ D \end{matrix}$$

$$\dot{y}$$

chrst d zst ! clal stch **P**

P

d st z zst !

E

A P E

D

F
T^c

L

T

stzchst z

ft d z dch ft clalrst zst zst

z clal
D
z clal st z clal

zst **F**

L

st r st r

T

T^C

zst rdst zst st

F̃

D̃

ch d st zst

F

ch d st zst

cllr d d

L

z st zst d st zst

T

r rst st st zst

T^C

zst

1.2. Related works

2. Framework of Trajectory Analysis

P A A P E

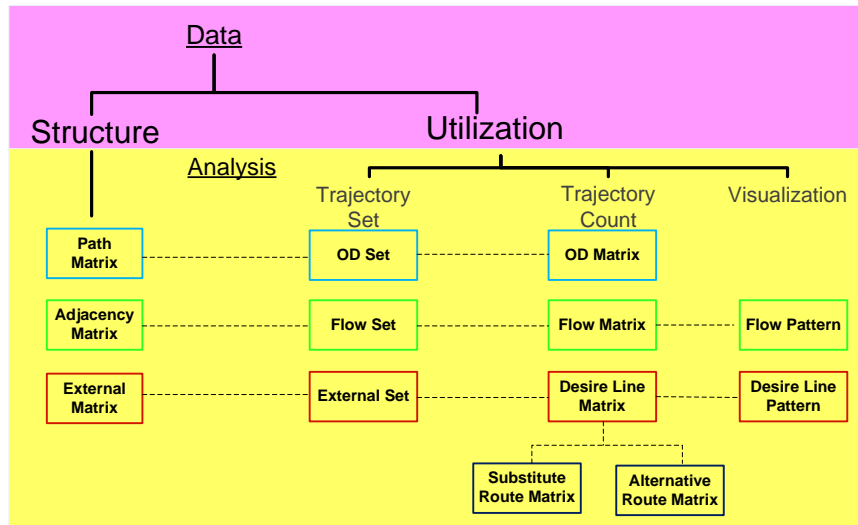


Figure 2. Framework of Ordinal Trajectory Analysis

stz d st

$$i \qquad j$$
$$S \quad t$$

$$\mathbf{A} \quad \mathbf{a}_{ij}$$

$$\mathbf{a}_{ij} = \begin{cases} 1 & \text{if } (i, j) \in E \\ 0 & \text{otherwise} \end{cases}$$

i

 j

$$\mathbf{P} \quad \mathbf{p}_{st}$$

S

 t

$$O \mid V \mid E \mid$$

$\mathbf{P} \quad \mathbf{p}_{st}$

$$\mathbf{p}_{st} = \begin{cases} (s, t) & \text{if there is a path from } s \text{ to } t \\ \text{otherwise} & \end{cases}$$

 (s, t)
$$S \quad t$$

$$\mathbf{P} = O(|V|^3)$$

$$\mathbf{A} \ \mathbf{P}$$

$$\mathbf{A} \ r \ \textit{catch} \ \ z \ \ d \ \mathbf{P}$$

$$\begin{array}{cc} st & st \\ st & \end{array}$$

$$t \ V_{\text{sink}} \ \mathbf{p}_{tj} \ , \ j \ t$$

$$\begin{array}{cc} r & r \end{array}$$

$$s \ V_{\text{source}} \ \mathbf{p}_{is} \ , \ i \ s$$

$$\begin{array}{ccc} & r & st \quad r \ st \\ st & r & \end{array}$$

$$\hat{\mathbf{p}}$$

$$\hat{\mathbf{p}}_{st} \colon \ 0 \ \mathbf{p}_{st}$$

$$\mathbf{E} \colon \ \mathbf{P} \ \mathbf{A}$$

$$\begin{array}{ccc} \mathbf{E} & \mathbf{P} & \mathbf{E} \ \mathbf{P} \end{array}$$

$$\hat{\mathbf{E}} \qquad \mathbf{E} \qquad \mathbf{E}$$

$$\hat{\mathbf{e}}_{ij} : \hat{\mathbf{p}}_{ij} \quad \mathbf{a}_{ij} : 0 \quad \mathbf{e}_{ij}$$

Proposition-1

$$\begin{array}{ccccccc} & & & & \mathbf{A} & & \hat{\mathbf{P}} \\ & & & & & & \\ & & \hat{\mathbf{E}} & & & & \\ & & \hat{\mathbf{E}} & \hat{\mathbf{P}} \circ \hat{\mathbf{E}} & & & \\ & & \mathbf{A} & \hat{\mathbf{P}} \circ \mathbf{A} & & & \\ & & \hat{\mathbf{E}} & & \mathbf{A} & & \\ & \hat{\mathbf{e}}_{ij} \circ \hat{\mathbf{p}}_{ij} = 0 & \hat{\mathbf{e}}_{ij} = 1 & \hat{\mathbf{p}}_{ij} & & \hat{\mathbf{e}}_{ij} = 0 & \\ & & \mathbf{p}_{ij} = 0 & \mathbf{e}_{ij} & \mathbf{p}_{ij} & \mathbf{a}_{ij} = 0 & \mathbf{p}_{ij} = 0 \\ & \hat{\mathbf{e}}_{ij} \circ \hat{\mathbf{p}}_{ij} = \hat{\mathbf{e}}_{ij} & & & & \mathbf{e}_{ij} \circ \mathbf{p}_{ij} = 1 & \end{array}$$

4. Network Utilization

4.1. Flow Matrix from Trajectory

$$\tilde{\mathbf{f}}_{ij} \quad \tilde{\mathbf{F}}_{ij}$$

$$\tilde{\mathbf{f}}_{ij} : tr \in Tr \mid tr = v_1, v_2, \dots, v_h \quad i, v_{h-1} \quad j, \dots, v_k \quad h, 1 \quad h \quad k$$

Tr

Algorithm 1: Trajectory Set to Flow Matrix-Set

Input: Set Tr of ordinal graph trajectories

Output: Flow matrix-set $\tilde{\mathbf{F}}$

$$\begin{aligned} & \tilde{\mathbf{f}}_{ij} \\ & d \quad st \\ & dz \quad stz \quad d \quad st \quad tr \quad Tr \\ & dz \quad ch \quad d \quad st \quad ij \quad ij \quad tr \\ & \tilde{\mathbf{f}}_{ij} \quad \tilde{\mathbf{f}}_{ij} \quad tr \\ & d \quad st \\ & d \quad st \\ & d \quad st \quad \tilde{\mathbf{F}} \end{aligned}$$

$$\dot{y}$$

$$\mathbf{F}$$

$$\mathbf{F}:\left|\tilde{\mathbf{F}}\right|$$

$$\mathbf{F}$$

$$\widehat{\mathbf{F}}:\mathbf{F}=0$$

$$\widehat{\mathbf{F}}$$

$$\widehat{\mathbf{F}}=\mathbf{A}$$

$$\textbf{Proposition-2}$$

$$\mathbf{F}$$

$$\mathbf{A}$$

$$\begin{array}{l} \tilde{\mathbf{F}}=\mathbf{A}\circ\tilde{\mathbf{F}}\\ \mathbf{F}=\mathbf{A}\circ\mathbf{F}\\ \widehat{\mathbf{F}}=\mathbf{A}\circ\widehat{\mathbf{F}} \end{array}$$

4.2. Generalized OD matrix from Trajectories

$$\tilde{\mathbf{D}}_{st} = \sum_{tr \in Tr} \sum_{v_1, v_2, \dots, v_p} \sum_{s, \dots, v_q} \sum_{t, \dots, v_k} \sum_{p, q, 1 \leq p \leq q \leq k} \tilde{\mathbf{d}}_{st}^{tr, v_1, v_2, \dots, v_p, s, \dots, v_q, t, \dots, v_k, p, q, 1 \leq p \leq q \leq k}$$

$$\tilde{\mathbf{D}}_{st} = \sum_{ij} \tilde{\mathbf{d}}_{st}^{ij}$$

Algorithm 2: Trajectories to OD Matrix-Set
Input: Set Tr of ordinal graph trajectories
Output: OD matrix-set $\tilde{\mathbf{D}}$

$$\tilde{\mathbf{D}}_{st} = \sum_{tr \in Tr} \sum_{v_1, v_2, \dots, v_p} \sum_{s, \dots, v_q} \sum_{t, \dots, v_k} \sum_{p, q, 1 \leq p \leq q \leq k} \tilde{\mathbf{d}}_{st}^{tr, v_1, v_2, \dots, v_p, s, \dots, v_q, t, \dots, v_k, p, q, 1 \leq p \leq q \leq k}$$

$$\tilde{\mathbf{D}}_{st} = \sum_{ij} \tilde{\mathbf{d}}_{st}^{ij}$$

$$\tilde{\mathbf{D}}_{st} = \sum_{ij} \tilde{\mathbf{d}}_{st}^{ij}$$

$$\tilde{\mathbf{D}} \quad \tilde{\mathbf{d}}_{st}$$

$$\tilde{\mathbf{d}}_{tj} \quad , \quad j \quad t \quad V_{\text{sink}} \quad st$$

$$\tilde{\mathbf{d}}_{is} \quad , \quad i \quad s \quad V_{\text{source}} \quad r$$

$$\mathbf{D}: \left| \tilde{\mathbf{D}} \right|$$

Proposition-3

$$\tilde{\mathbf{F}} \quad \tilde{\mathbf{D}}$$

Corollary-1

$$\mathbf{F} \quad \mathbf{D}$$

$$\widehat{\mathbf{D}}: (\mathbf{D} \rightarrow 0)$$

$$\widehat{\mathbf{D}} \quad \widehat{\mathbf{P}}$$

$$\widehat{\mathbf{D}} \quad \widehat{\mathbf{P}}$$

Proposition-4

$$\mathbf{D}$$

$$\mathbf{P}$$

$$\widehat{\mathbf{P}}$$

$$\mathbf{D} \quad \widehat{\mathbf{P}} \circ \mathbf{D}$$

$$\mathbf{d}_{st} \rightarrow 0 \qquad \mathbf{d}_{st} \rightarrow 0 \qquad \mathbf{d}_{st} \rightarrow 0 \qquad \widehat{\mathbf{p}}_{st} \circ \mathbf{d}_{st} \rightarrow 0$$

$$\mathbf{d}_{st} \rightarrow 0 \qquad \widehat{\mathbf{d}}_{st} \rightarrow 1$$

$$\widehat{\mathbf{p}}_{st} \rightarrow 1$$

$$\widehat{\mathbf{p}}_{st} \circ \mathbf{d}_{st} \rightarrow 1 \circ \mathbf{d}_{st} \rightarrow \mathbf{d}_{st} \qquad \mathbf{d}_{st} \quad \widehat{\mathbf{p}}_{st} \circ \mathbf{d}_{st}$$

4.3. Indirect Flow from Trajectories

$$r \rightarrow st$$

$$r \rightarrow st$$

$$\widetilde{\mathbf{L}}$$

$$\widetilde{\mathbf{l}}_{st} : tr \rightarrow Tr | tr \rightarrow v_1, v_2, ..., v_p \rightarrow s, ..., v_r, ..., v_q \rightarrow t, ..., v_k \rightarrow p, q, r, 1 \rightarrow p \rightarrow r \rightarrow q \rightarrow k \text{ .}$$

$$st$$

$$r$$

$$st$$

$$\tilde{\mathbf{F}} \qquad \qquad \qquad \tilde{\mathbf{L}} \qquad \qquad \qquad \tilde{\mathbf{D}}$$

$$\tilde{\mathbf{D}} \quad \tilde{\mathbf{F}} \quad \tilde{\mathbf{L}}$$

$$\tilde{\mathbf{L}} \quad \tilde{\mathbf{D}} \quad \tilde{\mathbf{F}}$$

$$\mathbf{L}: \left| \tilde{\mathbf{L}} \right|$$

$$\begin{matrix} \mathbf{D} & \mathbf{F} & \mathbf{L} \\ \mathbf{L} & \mathbf{D} & \mathbf{F} \end{matrix}$$

4.4. Alternative Route Flow

$$\begin{matrix} z & \cancel{st} & z & \cancel{st} & d & \cancel{st} \\ & & r & & & \end{matrix} \qquad \qquad \qquad \begin{matrix} \cancel{str} & r & \cancel{st} & \cancel{st} & \cancel{st} \\ & & & & \end{matrix} \qquad \qquad \qquad \begin{matrix} \mathbf{T} \\ \\ \mathbf{T}^c \end{matrix}$$

$$\begin{matrix} \tilde{\mathbf{L}}: & \tilde{\mathbf{T}} & \tilde{\mathbf{T}}^c \\ \mathbf{L} & \mathbf{T} & \mathbf{T}^c \end{matrix}$$

$$\mathbf{T}$$

Proposition-5

$$\begin{matrix} \tilde{\mathbf{T}} & \mathbf{A} \circ \tilde{\mathbf{L}} \\ \mathbf{T} & \mathbf{A} \circ \mathbf{L} \\ \hat{\mathbf{T}} & \mathbf{A} \circ \hat{\mathbf{L}} \end{matrix}$$

$$\begin{array}{llll}
 \mathbf{a}_{ij} & 0 & \tilde{\mathbf{t}}_{ij} & \tilde{\mathbf{L}} \\
 \mathbf{a}_{ij} & 1 & \tilde{\mathbf{t}}_{ij}^c & \tilde{\mathbf{L}} \quad \tilde{\mathbf{T}} \\
 \mathbf{a}_{ij} \circ \mathbf{l}_{ij} & 1 \circ \mathbf{l}_{ij} & \mathbf{l}_{ij} \quad \mathbf{t}_{ij} & \tilde{\mathbf{t}}_{ij} \quad \mathbf{a}_{ij} \circ \tilde{\mathbf{l}}_{ij}
 \end{array}$$

5. Relationships among matrices for Network Structure and Network Utilization

5.1. Relationship between Flow and Origin-Destination

Proposition-6

$$\begin{array}{llll}
 \mathbf{D} & \mathbf{d}_{st} & \mathbf{F} & \mathbf{f}_{ij} \\
 \mathbf{A} & \mathbf{a}_{ij} & & \\
 \mathbf{F} & \mathbf{A} \circ \mathbf{D} & \mathbf{D} & \\
 \mathbf{a}_{ij} & 1 & \mathbf{a}_{ij} \circ \mathbf{d}_{ij} & 1 \circ \mathbf{d}_{ij} \quad \mathbf{d}_{ij} \\
 ij & & i & j \\
 \mathbf{d}_{ij} & 0 & \mathbf{a}_{ij} & 0 & \mathbf{a}_{ij} \circ \mathbf{d}_{ij} & \mathbf{d}_{ij} \\
 \mathbf{a}_{ij} & \mathbf{A} \circ \mathbf{D} & \mathbf{D} & \\
 \mathbf{F} & \mathbf{D} & \\
 \mathbf{A} & \mathbf{A} \circ \mathbf{F} & \mathbf{A} \circ \mathbf{D} & \mathbf{F} & \mathbf{A} \circ \mathbf{D} \\
 \mathbf{D} & \mathbf{F} & \mathbf{A}
 \end{array}$$

Proposition-7

$$\begin{array}{ll}
 \mathbf{T} & i \\
 j &
 \end{array}$$

$$F \quad T \quad A \circ D \quad D \quad T^C$$

$$T \qquad \qquad \qquad F$$

$$\text{Corollary-2} \qquad \qquad \qquad F \quad T$$

$$\begin{matrix} F & A \circ D & T \\ T & A \circ D & F \end{matrix}$$

$$\begin{matrix} & T & & i \\ j & & z \ z \ z \ d & ij \\ T \ 0 & & & F \ A \circ D \end{matrix}$$

$$T^C$$

$$\text{Proposition-8} \qquad \qquad T^C \qquad \qquad \qquad i$$

$$T^C \quad \widehat{E} \circ D$$

$$\begin{matrix} & A \circ D & D & T^C & & T^C & D & A \circ D \\ T^C & \widehat{P} \circ D & A \circ D & & D & & & \\ & T^C & \widehat{P} & A \circ D & \widehat{E} \circ D & & & \end{matrix}$$

5.2. Identity Relationships

Theorem-1

$$\begin{matrix} & \hat{E} & & A & & \hat{P} \\ L & & & & & F & & D \\ & & & T & & & & \end{matrix}$$

$$\begin{matrix} L & T & \hat{E} \circ D \\ A \circ D & D & \hat{E} \circ D \\ D & \hat{P} \circ D \\ F & A \circ D & T \end{matrix}$$

$$\begin{matrix} L & T & \hat{E} \circ D & & D & F & T & \hat{E} \circ D \\ & F & T & D & \hat{E} \circ D & & A \circ D & D & \hat{E} \circ D \\ A \circ D & D & \hat{E} \circ D & D & (\hat{P} & A) \circ D & & A \circ D \\ A \circ D & D & \hat{P} \circ D & A \circ D & D & \hat{P} \circ D & & F & L & \hat{P} \circ D \\ & & & & F & L & \hat{E} & A & \circ D \\ F & L & \hat{E} \circ D & A \circ D & & F & L & T^C & A \circ D \\ & & F & T & T^C & T^C & A \circ D \\ F & A \circ D & T & & & F & D & \hat{E} \circ D & T \\ & & & & & L & T & \hat{E} \circ D \end{matrix}$$

| Valid for all instances | Valid only for instances involving fully utilized networks |
|---|---|
| $\hat{E} \quad \hat{P} \circ \hat{E} \quad A \quad \hat{P} \circ A$ | |

| | |
|--|-------------------------|
| $\hat{F} \quad A \quad F \quad A \circ F$ | $\hat{F} \quad A$ |
| $\hat{D} \quad \hat{P} \quad D \quad \hat{P} \circ D$ | $\hat{D} \quad \hat{P}$ |
| $\tilde{F} \quad \tilde{D} \quad F \quad D \quad \hat{F} \quad \hat{D}$ | |
| $\tilde{T} \quad A \circ \tilde{L} \quad T \quad A \circ L \quad \hat{T} \quad A \circ \hat{L}$ | |
| $F \quad T \quad A \circ D \quad D \quad T^C$ | |
| $T \quad A \circ D \quad F \quad T^C \quad \hat{E} \circ D$ | |
| $L \quad T \quad \hat{E} \circ D$ $A \circ D \quad D \quad \hat{E} \circ D$ $D \quad \hat{P} \circ D$ $F \quad A \circ D \quad T$ | |

6. Conclusions

7. End Notes

$$\mathbf{F} = \mathbf{A} \circ \mathbf{D}$$

$$\mathbf{D} = \mathbf{F} / \mathbf{A}$$

$$D_{st} = \begin{cases} 0 \\ A_{st} & \begin{matrix} 1 \text{ if } F_{st} / D_{st} = 1 \\ 0 \text{ if otherwise} \end{matrix} \end{cases}$$

$$A_{st} = 0$$

$$\hat{\mathbf{L}} \quad \hat{\mathbf{D}} \quad \hat{\mathbf{F}}$$

$$r \; st$$

$$st$$

$$\hat{\mathbf{T}}$$

$$r \; st$$

Acknowledgement

References

34

$z\ r\ \cancel{s}st\ drdz\ z\ st$ 25

$z\ z\ r\ \cancel{s}st\ D\ ft\ dd\ ft$ 130
 $z\ r\ \cancel{s}st\ dst\ ij\ z\ rr$

$d\ ftdr$ 17
 $z\ r\ \cancel{s}st\ drdz\ z\ st\ D\ d\ ft\ ft$
 $r\cancel{s}d\ z\ r$

27

$z\ z\ r\ \cancel{s}st\ D\ ft\ dd\ ft$ 131

136

$z\ z\ r\ \cancel{s}st\ D\ ft\ dd\ ft$

$z\cancel{s}zrdz\ c\cancel{r}D\ d\ s\cancel{r}r\cancel{s}\cancel{d}\ r\ zst\ r$

$DDD\ R$

$z\ c\cancel{h}z\ d\cancel{c}h\ z\ r\ \cancel{s}st$ 45

$zst\ r\ std$ 5

z r st d st z

ddch fr std ijr d d st z s s d z r r
dfz st d z st dr z ch z d r J dz fd z

z

z r s st D ft dd ft 126

s d z st z

z T z R d dr 12

z z r s st D ft dd ft 134

DDD z rz st r s d fd st z r s st R r s d r 10

st ft st R z st

z d st dr

z chz dch z r s st 45

ddch fr std 8 7 Rd ch s d z st z d d d s d cl d ft
z chR z st

z r s st drdz z st d st ch ft z 30

ddch fr std 3 st s d z st z
d d d std z st r chz dch d ft z r s st

z

z r st d ftz 16

cl d ft z r st

s d fd st z r s st

R r s d r 12

z ~~st~~ ft st z r ~~st~~ R r ~~st~~ r 12

z r ~~st~~ st

drdz z st d st ch ft z 21

ddch fr ch ~~st~~ z st z d d d d st d d z st d
z st r! Rd dr z ch d ftdr!

st ~~st~~ z st z d d d st d D z r ~~st~~ r z R d st z r ~~st~~ st R st ch dr ddch fr st d

z st d 9

d ftz z z st R r ~~st~~ r!

131

z r ~~st~~ st

drdz z st d st ch ft z 29

ddch fr st d 7 st R R J ~~st~~ z st z d d d chz dr d ftz
z st R r ~~st~~ r

7 st ~~st~~ z st z d d d

T st r st ft

DDDR r ~~st~~ r z ch z st r