PEDESTRIAN ROUTE CHOICE SELF-ORGANIZATION

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Abstract

In this paper, we describe a new approach for modelling route choice in pedestrian dynamics simulations. The model is stated in the mesoscopic paradigm using a queuing network graph based on agents' local decision. Contrary to traditional models, route choice here is not given exogenously but emanates as a consequence of the prescribed behavioural patterns of the various agents. Consequently, the route choice probability and travel time are outputs to the simulations rather than inputs. By "route choice self-organization (RCSO)" we denote the phenomenon that agents autonomously and endogenously during the simulation decide to change their plans with respect to route choice. The model is illustrated in a demonstration example in a network graph.

Keywords: route choice self-organization (RCSO), sink propagation value (SPV), dynamic traffic assignment, pedestrian dynamics, multi-agent

INTRODUCTION AND RELATED WORK

In this paper, we describe a new approach for modelling route choice in pedestrian dynamics simulations to produce route choice self-organization (RCSO). By "route choice self-organization (RCSO)", we denote the phenomenon that occurs in situations of medium to high density of pedestrians where some pedestrians will accept deviations in order to avoid crowded situations arising on the shortest route. This phenomenon is often observed in real crowded scenarios.

The multi-agents pedestrian movement model proposed in this paper is stated in the mesoscopic paradigm using a queuing network graph, which contains a regular lattice as a special case. In contrast to conventional traffic assignment methods (see e.g. Ortuzar and Willumsen, 2001) that assign pedestrians to specific routes (either precalculated or continuously updated during the simulation) based on expected travel times, our model allows for an autonomous choice of the route of the single agents during runtime based on local information. Thus, the route choice probability as well as the travel time are the outputs of the simulation rather than the inputs. Route choice hence emanates as a consequence of the prescribed behavioural patterns of the various agents.

In the model, walkable space is represented using a directed network graph, where the cells of the lattice constitute the vertices and the neighbourhood interconnections between the cells the edges. The regular lattice used e.g. in cellular automata models is a special case. The extension of each cell in the network graph may range from a small grid as typically used in cellular automata models (e.g. 0.4m square) to broad cells (e.g. 4 m square). The movement of the pedestrian agents is governed by two levels of detail:

- Movement within the cell follows a macroscopic description based on the fundamental diagram.
- Once the pedestrian hits the boundary of the cell, she has to choose the next cell. This choice is the core of the model and assumed to be made based on a local trade-off between the navigation (i.e. getting closer to the exit) and crowdedness of the adjacent cells.

Thus, the modeller needs to supply a fundamental diagram and the parameters governing the local trade-off between choosing the shortest path and unease for crowded situations while the outcome of the models is the flow on the links of the network graph (from which route choice can be obtained) as well as the movements of each single pedestrian.

The proposed model is an extension of the mesoscopic model presented in Teknomo & Millonig (2007) in various aspects. While Teknomo & Millonig (2007) only considers a regular grid, in this contribution a general network graph is allowed for, including multiple edges connecting two cells. Furthermore the emphasis in this paper is on route choice rather than navigation, i.e. dealing with situations where many pedestrians try to find their way in an environment they are familiar with, while Teknomo & Millonig (2007) dealt with pathfinding for single pedestrians in unfamiliar territory. Depending on the choice of the network graph and the parameters, the model falls into the microscopic, the macroscopic or the mesoscopic level. In the extreme case, the whole space is modelled as only one cell and the movement is fully governed by the fundamental diagram. Such a model clearly is macroscopic as the interaction of pedestrians is modelled using flow-density relations. In this respect the model can be seen as an extension of the queuing network model proposed by Lovas (1994).

On the other end of the scale using cells with a small equivalent length such that per time step one cell is traversed by each pedestrian and the maximum space capacity of the cells is equal to one pedestrian the main component of the model is the decision of the next cell to enter and the model is considered to be microscopic. In this case the model can be viewed as an extension to the cellular automata model documented in the literature (see e.g. Blue and Adler (2000), Kretz and Schreckenberg (2006), Schadschneider (2001)) by adding on-route route choice capabilities to the agents.

Finally using a regular grid with a side length of 1m to 4m, for instance, we obtain a mesoscopic pedestrian simulation model where each cell may be occupied by a number of pedestrians. In contrast to the model of Florian et al (2001) and Hanisch et al (2003) the model presented in Teknomo & Millonig (2007), on which the proposed model is based, represents each pedestrian as an individual agent. Furthermore, in this mesoscopic model, not only the pedestrian flow is modelled but also every pedestrian is represented as an intelligent agent who keeps the timing of entering and leaving a cell. Mesoscopic models achieve numerical superiority in comparison to microscopic models by introducing simplifications and omitting details. In particular, detailed pedestrian movement in continuous space and detailed pedestrian behaviour (e.g. collision avoidance behaviour) are omitted.

The paper is organized as follows: The next section provides a detailed description of the proposed model including a discussion of the concept of sink propagation value (SPV). Before the conclusion, we illustrate the model using a simple demonstration example.

THE PROPOSED MODEL

In this section, the proposed model is discussed. In the model, the walkable space is represented using a three-dimensional non-planar directed multi-graph as it may contain multiple edges (e.g. if there are multiple routes connecting different vertices such as stairs and escalators in parallel). The edges correspond to cells partitioning the walkable space.

Definition 1: The set of *followers* of vertex v, denoted by $\Gamma(v)$, is the set of *adjacent* edges that are *incident from* vertex v. The *predecessors* (denoted by $\Gamma^{-1}(v)$) are the sets of adjacent edges *incident to* vertex v. By *neighbourhood* of a vertex we denote the set of all vertices that are connected to a vertex including itself.

Each vertex in the graph is either contained in a basin (representing entry points, exit points or service counters) or a mere connection point between edges. There is no limitation on the number of agents that can be accommodated within a vertex. Each edge in the graph represents real space such as rooms, doors, or facilities such as stair, ramp, elevator, and escalator, etc. Therefore, space constraints occur which is formularized in the following definition:

Definition 2: The **space capacity** of an edge $\overrightarrow{i\,j}$ connecting vertices i and j is the product of equivalent length $\ell_{\overline{i\,j}}$, equivalent width $\omega_{\overline{i\,j}}$ and agents' perception on maximum density $\rho_{\max}$.

$$c_{\overline{i\,j}} = \ell_{\overline{i\,j}}\omega_{\overline{i\,j}}\rho_{\max} \tag{1}$$

Here the notion "equivalent width" and "equivalent length" are chosen because they may differ from actual width and length but represent other impedance factors rather than mere distance. Note in particular that the maximum capacity is hence influenced by the pedestrians' properties. This makes it possible to model different perceptions of density for different agents. Space capacity, therefore, is a perceived capacity as apparent to the agents. The pedestrians are modelled as individual entities that move through the network graph.

Definition 3: An **agent** is an autonomous discrete entity (i.e. pedestrian, vehicle, goods) that moves based on local behavioural rules from a cell contained in a set of starting cells (denoted as **source basin**) to a cell within a set of cells (denoted as **sink basin**) where his journey ends via visiting a number of intermediate sets of cells where activities take place (called **saddle basin**).

Note that we use the notation "basin" rather than origin and destination as the term origin is used to refer to a single region where the agents start their journey while destination is used for a single region where the agents end their journey. In our notation, a source basin may consist of several not necessarily connected vertices such as e.g. different doors leading to the same street.

Agent's movement is directed from her origin vertex contained in a source basin to her destination vertex contained in a sink basin. Different agents might correspond to different source-sink-pairs while the presentation below will always be given from the perspective of one agent and hence refer to fixed OD-pair. The motion of pedestrians is modelled using discrete time steps by specifying behavioural updating rules in order to progress in simulation time. Movement within the cell is governed by a fundamental diagram which either can be supplied by the user from external sources or included in the model as follows: current speed v_t is adjusted based on current link density ρ_t (number of pedestrians inside the cell) at $t = t_{in}$ (the time the agent enters the cell) and a speed density relationship given as $v_t = f(\rho_t)$. Let $\rho_{\vec{ij}} / \rho_a^{\max}$ be the *normalized density* based on agents' perception of maximum density, then

$$v_t = v_{\max}\left(1 - \mathrm{BetaCDF}\left(\frac{\rho_{\overline{i\,j}}}{\rho_a^{\max}};\varsigma,\tau\right)\right) \tag{2}$$

where $v_{\max}$ denotes the maximal speed for the agent. Here BetaCDF denotes the cumulative distribution function of the beta-distribution, depending on two positive real parameters ς, τ in $(0,1]$. The beta-distribution is chosen due to its flexibility and the fact that it is supported on (0,1]. The parameters of Beta distributions can take any positive real value (i.e. range in $\Re^+$) but values in this range do not add any reasonable fundamental diagram hence we restrict the range. Using this velocity the time it takes to traverse a cell of equivalent length $\ell_{\overline{ij}}$ equals $t_{out} = \ell_{\overline{ij}} / v_t$. Since the model proceeds with a fixed time step dt the actual time of leaving the cell, denoted as t^* is computed as

$$t^* = \lceil t_{in} + t_{out} \rceil \tag{3}$$

Here the symbol $\lceil . \rceil$ denotes the smallest real such that $t^* = t_{in} + kdt$ for some integer k where t_{in} denotes the time the agent enters the cell.

When leaving a cell and thus reaching a vertex, the agents decide which edge to enter next. This decision is taken autonomously based on a set of rules using the agent's observation of the local environment. In our model, the agent's sensing ability is limited to observe only the followers density and space capacity. Additionally the agents have complete information of a notion of distance to their assigned sink at each vertex, which is discussed below. The decision on which edge to enter next is determined by the interplay of four factors:

1. **Permission**: Here the permission value is a binary quantity indicating whether an agent is allowed to enter a certain edge at a specified discrete time t. It is determined by the connections present in the network graph, which might be temporarily closed. The permission value for a dynamic environment (e.g. door open and closed) can be obtained through interactive removal and addition of edges during the simulation running time and is represented by the set $\Gamma(i,t)$ containing only those edges inside the followers of vertex i that are permitted at time t.
2. **Interaction** between pedestrians is represented by a function of edge density. If the edge density is high, the attractiveness to go to that edge is reduced. Suppose the agent's position is in vertex i and let j be an edge contained in the followers (by convention the edge connecting a vertex with itself is always contained). Let $\rho_{\overline{ij}}$ be the current density of edge $j \in \Gamma(i)$ and $c_{\overline{ij}}$ the maximal capacity of edge j. Then the interaction weight at edge j is given as

$$I_{\overline{ij}} = 1 - \mathrm{BetaCDF}(\frac{\rho_{\overline{ij}}}{c_{\overline{ij}}}; \phi, \varphi) \tag{4}$$

   Again, we are using the cumulative distribution function of the beta-distribution $\mathrm{BetaCDF}(x;\alpha,\beta)$. Here, $\rho_{\overline{ij}} / c_{\overline{ij}}$ is the *edge density ratio.*
3. **Navigation** is represented by the concept of *sink propagation value* (SPV) discussed in more detail in the next subsection. For the moment, it suffices to say that the SPV can be seen as a function measuring the distance of the vertices to

the sink basin. Let v_i and v_j be the sink propagation values of the current vertex and the vertex reached via the edge j in the followers respectively. Then the normalized SPV difference at the current vertex is defined as follows:

$$N_{ij} = BetaCDF\left(\max\left(\frac{v_i - v_j}{\max_{k\in\Gamma(i,t)}\{v_i - v_k\}}, 0\right); \vartheta, \theta \right), j \in \Gamma(i,t) \tag{5}$$

This definition makes sense, whenever the denominator is strictly positive which will always be the case outside of the sink basin.

4. Finally, the **dynamics** enter the decision problem via the temporal dependence of the interaction and the permission term.

The decision then is done deterministically based on maximization of the attractiveness (modelled as the product of the interaction and the navigation term) of the edge: The selected edge k is given by

$$k = \arg\max_{j\in\Gamma(i,t)} I_{ij} N_{ij} \tag{6}$$

In cases of a draw (i.e. two equally attractive edges), randomly an edge amongst the edges of best SPV improvement within the set of optimal attractiveness is chosen. Although the interaction term should prevent collisions (i.e. agents wanting to move into cells which are already full) this may happen for some choices of parameter values (e.g. if the navigation part gets a high weight). Hence, if an agent tries to enter an already full cell, she is stopped and remains in her current cell.

Thus the pedestrian simulation model has six parameters to be calibrated which are ϕ and φ for the interaction, ϑ and θ for navigation, ς and τ for the speed-density relationship. Note that the fundamental diagram is the input of the model (through speed-density relationship) rather than the output.

This decision rule is based on two types of knowledge: navigation represents global information on a general distance function whereas the interaction term indicates local information on pedestrian densities. The global information includes full information regarding the network while the local information covers only a vertex and its neighbourhood. Global information is based on a function of sink propagation value while the local information is a function of the edge's density. Both information is stored and accessed locally. At a decision vertex, each agent only requires to gain information at that particular neighbourhood of the vertex. The decision to move from one cell to the other cell is assumed to only take place in vertices. The decision on which edge to enter in the next step incorporates a local trade-off between getting "closer" to the exit (where the particular notion of closeness is represented in the navigation term) and the urge to avoid dense regions. The weights in this trade-off are implicitly influenced by the choice of the parameters in the beta-cumulative distribution functions.

Sink Propagation Value

With respect to navigation, we use a concept called "sink propagation value" (SPV). SPV is a function (monotonically increasing with increasing minimum distance from the sink as implied by the network graph) assigning a value to each vertex that is implementing a general notion of distance from the sink. The name derives from the fact that the calculation of the SPV typically is done by propagating the values from the sink node into all other connected nodes.

Definition 4: A function defined at each vertex of a directed network graph (where there exists a set of strictly positive values attached to the edges incorporating the notion of a distance between the vertices) is called a *Sink Propagation Value* (SPV) if the following properties hold:

1. Positivity: $v_i \geq 0$.

2. Zero at sink basin: $v_s = 0$ for each vertex s contained in the sink.

3. Infinity if the sink is unreachable from the vertex: $v_i = \infty \Leftrightarrow i \not\to s$

4. Strictly monotonically increasing with increasing shortest distance from the sink: $v_i > v_j \Leftrightarrow \inf_s d_{is} > \inf_s d_{js}$, where d_{is} is the shortest distance from vertex i to vertex s contained in the sink and the infimum is taken over all vertices contained in the sink.

Various different SPV concepts can be obtained implicitly by using computational methods such as reinforcement learning (i.e. Q Learning), the Bellman flooding algorithm and the so-called distance transform. In this sense, the SPV represents global information of the network distance function that is stored locally at the vertices. Thus, we can also view the SPV as a transformation of the network global information into local information at each vertex. For a dynamic environment with many doors that potentially can be opened and closed during the simulation, every different configuration of open and closed doors leads to another SPV. For a large number of possible configurations pre-computation of all SPVs is computationally infeasible. Navigation algorithms for such dynamic environments can be found in Teknomo & Millonig (2007).

Route Choice Self-Organization

By route choice self-organisation (RCSO) we denote the phenomenon that the agents in the simulation deviate from the shortest path autonomously and endogenously during the simulation choose different routes in order to avoid dense regions. In this section, we discuss why the model proposed in the last section is capable of exhibiting this phenomenon.

The RCSO emanates in relatively dense scenarios where the optimal route in terms of walking length is abandoned by pedestrians due to their preference to avoid crowded edges, i.e. in a sense pedestrians react to emerging congestions and implicitly they take the corresponding deviations into account. This can be demonstrated using a comparative static analysis analysing the choice between two alternative routes: At low levels of pedestrian flow, the navigation term will dominate the interaction term. Due to the navigation criterion to select the shorter route, edges that correspond to the shorter routes will be filled first. Increasing the flow level, the density on the shorter route will increase making the interaction term more important and the corresponding edge less attractive. Consequently, more pedestrians will choose the longer route to avoid regions of high crowd density. Increasing the flow levels even more up to a level where also the alternative route is congested the navigation again gains importance. This model conforms qualitatively to observations in real world scenarios.

The choice of route occurs in the model as a self-organization phenomenon i.e. it is neither modelled explicitly nor controlled centrally but instead emerges as a consequence of the autonomous optimizations of the agents based on their sensing ability. The optimization is performed including global and local information. The global information guides agents' navigation based on the SPV. Hence, it is assumed that the agent has perfect information on the infrastructure. The interaction on the other hand uses only local information represented as a generalized cost function of the perceived density on the edges adjacent to the decision vertex where the agent currently is located.

ILLUSTRATIVE EXAMPLE

The proposed model is illustrated in a simple model where it is demonstrated that contrary to conventional dynamic travel assignment (DTA) algorithms to provide route choice proportions and route flow of traffic assignment, the proposed method has no problems with complicated nested routes. For the numerical illustration, we use the following network graph: The network is illustrated on the left (where the distances between the vertices are provided in the edges and the numbers on the vertices are vertex identification numbers) and the corresponding sink propagation value (shortest path) of the network is shown in the right of Figure 1.

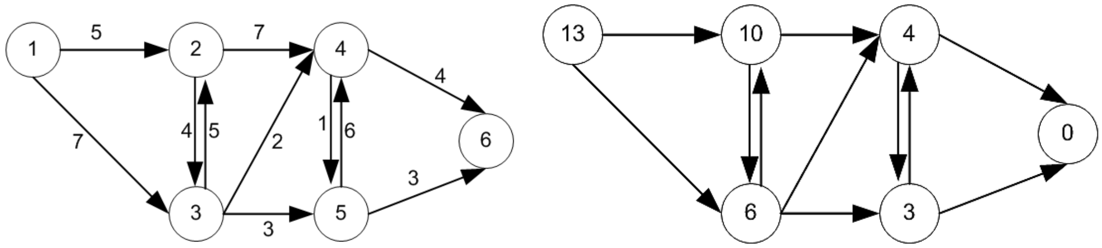

Figure 1: *Network example with distance (left) and SPV (right)*

The equivalent width is 0.999m for all links[1]. A hundred agents move from source vertex 1 to sink vertex 6 with a maximum speed of 1.2 meter/second and a maximum density of 4 pedestrians per square meter generated at once. The simulations are run with a discrete time step representing 1 second.

As presented above the model contains six parameters to be calibrated which are ϕ and φ for the interaction, ϑ and θ for navigation, ς and τ for speed-density relationship. These parameters are adjusted in order to minimize a criterion function. The details of the optimization are of no concern in this paper and will be discussed elsewhere. Note furthermore that there is no guarantee that there exists a unique global minimum. In fact, it will be seen below that this is not always the case. The main purpose of using different criterion functions is to show that different optimization strategies lead to different solutions. Viewing the parameters as different tradeoffs between minimizing path length and avoiding crowded regions this implies – maybe not surprisingly - that pedestrians need to use different tradeoffs in order to achieve different goals.

We are going to optimize the trade-off in terms of three criterion functions: Minimum egress time, minimum average travel time and shortest distance.

Table 1: *Parameter set for minimum egress time*

	ς	τ	ϕ	φ	ϑ	θ	**Egress time**	**Average Travel Time**	**Average Travel Distance**	**Average Travel Speed**	**Average Total Links**	**Total Route**
A	1	0.01	1	0.01	0.01	1	29	16.95	14.19	1.1889	3.36	8
B	1	0.01	1	1	0.01	0.01	29	16.91	14.23	1.1891	3.32	8
C	1	0.01	1	1	1	1	29	16.95	14.19	1.1889	3.36	8

Minimum Egress Time

Egress time is used in evacuation studies as the time span between the occurrences of the emergency until the last pedestrian exits the infrastructure. Ignoring the reaction time the illustration example shows a global minimum of egress time of 29 seconds for the 100 agents. The global minimum is achieved e.g. by the three sets of parameters shown in Table 1*:* that create three evacuation strategy named A, B and C as shown in Figure 2. Strategy A and C produce the same distribution of route choices. Note that the search domain is limited to [0.01, 1]. Not surprisingly in all cases ζ=1 and τ=0.01 are chosen. This implies that in the fundamental diagram, speed is close to its maximal value even for high levels of density. The choices of the weighting of the interaction and the navigation term assure that pedestrians always move and never want to enter edges that are already

[1] Equivalent width is chosen just below 1 meter in order for space capacity not to equal an integer quantity hence allowing velocity to drop to zero for density to reach 1.

full. Since the decisions of the agents are deterministic and chosen based on a maximization concept resulting in distinct choices it can be verified that in the vicinity (in the parameter space) of these choices the same qualitative behaviour prevails, while average travel speed can be increased to almost 1.2 m/s by lowering τ to a smaller positive value e.g. τ=0.001.

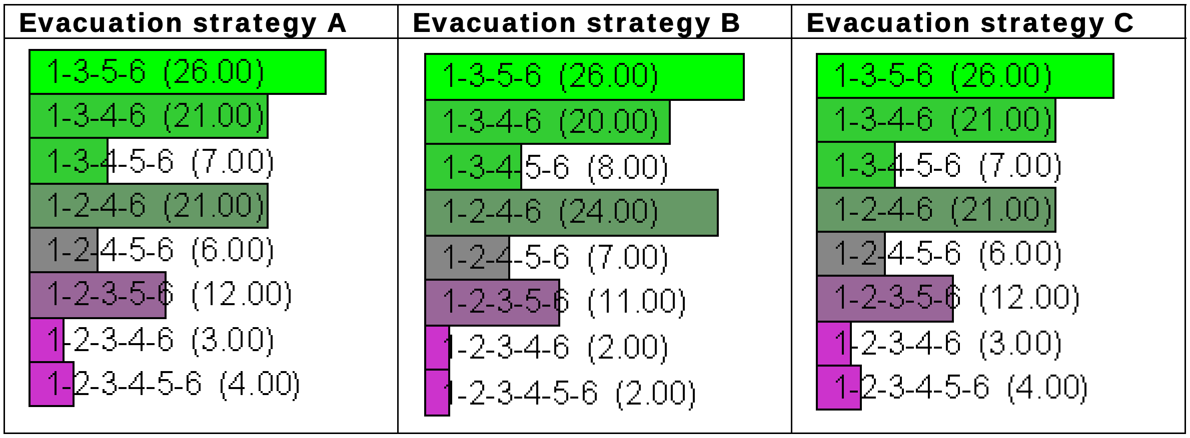

Figure 2: Evacuation route strategies to minimize egrees time

Table 2: Parameter for minimum travel time

ς	τ	ϕ	φ	ϑ	θ
1	0.001	0.065	1	1	0.065

Minimum Travel Time

The average travel time is measured from the first time an agent enters the first edge until she reaches the sink node for all agents. It is interesting to find out that the route choice for the minimum average travel time is not necessarily the same as minimum egress time. Setting the minimum average travel time produces only a single set of parameters that produce a global minimum of 15.75 seconds[2]. The parameter set produces a route strategy with most pedestrians using the path sequence 1-3-5-6. It has the egress time of 31 seconds, slightly higher than global minimum of egress time (29 seconds) and the average travel distance of 13.61 meter (which is also slightly higher than the shortest path). The average speed is 1.199 m/s and average total links is 3.03.

[2] The search is not exhaustive. The minimum travel time for a single agent would be shortest path divided by max speed, which is 10.83 seconds in this case.

Shortest Path

Calibration of the simulation to minimum average travel distance has produced many sets of parameters achieving the optimal result of all agents choosing the shortest path. A small sample of these parameter configurations is shown in Table 2. For most of these sets of parameters the route path sequence 1-3-5-6 is used. However, also the sequence 1-3-4-6 produces a route of equal length, which is frequented by pedestrians for some parameter configurations. Another possible shortest route (sequence 1-3-4-5-6) did not occur in any of the obtained models maybe due to the incomplete search.

Table 3: Set of parameters for minimum average travel distance

ς	τ	ϕ	φ	ϑ	θ	Egress time	Average Travel Time	Average Travel Distance	Average Travel Speed	Total Route
1	0.1	1	0.1	0.98	1	44	18.1	13	1.0246	1
0.57	0.1	0.53	0.1	1	0.1	52	21.55	13	0.92849	1
0.1	0.1	0.1	0.1	1	0.1	82	33.11	13	0.51145	1
1	**0.52**	**0.52**	**0.18**	**1**	**0.88**	**116**	**33.86**	**13**	**0.41952**	**2**
1	0.81	0.81	0.1	1	1	286	73.91	13	0.25646	1

Finally, it should be noted here that the parameters for the fundamental diagram are of no importance as they do not affect the route choice in this example but only the average travel time. Hence, in Table 3 the parameters ς and τ could be changed to arbitrary other values without changing the travel distance.

This example shows that for different criterion functions (egress time, minimum average travel time and minimum average travel distance) different parameter sets produce optimal results. While pedestrians have only limited capability to influence the fundamental diagram, they can influence the trade-off between navigation and interaction. The proposed model might be of help in modelling the trade-off usually employed and accordingly evaluate the design of the infrastructures.

CONCLUSION

The route choice self-organization can be seen as a new and novel alternative to existing multi-agent dynamic traffic assignment models. The proposed model is using a local trade-off between agent's navigation and interaction which is much simpler than traditional DTA for complex routes and it is useful when the detailed movement of pedestrians is in the focus of attention and the space is larger than a single room. At this level of detail, the interaction of pedestrians with obstructions is not considered to be important. The main output of the model is the dynamic flow from one spatial region (e.g. building, floor, or room) to another spatial region. The proposed model is flexible in the choice of the level of detail allowing for macro-, meso- and microscopic formulations. It also includes explicit models of navigation. The basic underlying paradigm of the model appears to have a more general appeal and might prove useful also for multi-agents models of vehicular traffic flow.

In a specific example, it is demonstrated that the choice of the parameters can be used in order to obtain different characteristics of the trade-off between short paths and avoiding dense crowds. In particular, the parameter sets in order to optimize various criteria such as egress times or average travel time are supplied. Hereby it was observed that the optimizing parameter sets are not unique. The illustrative examples also show that with a large number of agents (close to the capacity of the infrastructure) egress times are lower when the proposed route choice is used compared to the case where agents always adhere to the shortest path to exit the infrastructure. This is in contrast to the intuitive urge of evacuees to exit along the shortest routes.

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