Intersection Analysis Using the Ideal Flow Model

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***Abstract*—The optimization of traffic flow capacity utilization in an intersection often relies on historical traffic flow demand. Instead of relying on traffic demand, a novel model, called the ideal flow model, aids in this optimization by making use of traffic supply information. The model offers a unique approach to analyzing intersection capacity since it utilizes the network structure as well as the transition probability of a random walk or using Markov chain, as an alternative to the information that an OD matrix provides. In this paper, the ideal flow model is used to analyze a crossing type intersection. It investigates the steady state solution when the total flow is maximized and when the design of an intersection is optimum. Three clusters emerged from the simulations performed, and all three clusters depend on the ratio of the total inflow capacity to the total outflow capacity. The optimum design of the intersection is attained when the total inflow capacity is the same as the total outflow capacity. In this case, the total flow accommodated by the intersection reaches its maximum value.**[1]

I. Introduction

The rapid advancement of Intelligent Transportation Systems has led to the improvement of the utilization of road networks [1]. When determining the most efficient and reliable way of distributing traffic density across road networks, the usual method requires the expensive origin-destination (OD) survey. The OD survey is a study on current traffic demand to predict future traffic patterns [2]. A recent study [3] has proposed a traffic prediction model based on "big data transportation", where deep learning is applied. In the preceding cases, traffic assignment is done based on demand. Instead of looking at traffic patterns from the view of demand, in this paper, traffic flow modeling is seen from the perspective of supply. This alternative traffic flow model is called the **ideal flow model**, which aims to determine the most efficient utilization of a network. This happens when the flow is distributed uniformly over time and space. This idea is supported by [4] that proposed a "dynamic route guidance system" based on the Maximum Flow Theory which balances the traffic load on a road network. The actual traffic flow can be managed in the direction of the ideal traffic flow, which the ideal flow model provides. Thus, the model can aid in the provision of intelligent traffic information [5].

The approach of the ideal flow model is unique because the travel demand is represented by the use of the transition probability matrix of a random walk instead of the information from an OD matrix. In the micro level, if one views the distribution of the travel demand flow to any direction from an intersection as a probability distribution, aggregating these flows will result in a transition probability matrix that is also a stochastic matrix. The use of a stochastic matrix as an alternative to using the OD matrix yields some interesting properties, such as:

- scaling an ideal flow matrix by multiplying it by a positive constant produces an equivalent ideal flow;
- an ideal flow matrix is a *premagic* matrix, because of flow conservation on each node.

In this paper, the application of the ideal flow model is demonstrated by analyzing traffic flow on the most common type of intersection: a simple crossing type. The analysis will be done on a macroscopic level and at a steady state condition. Since the interest is on the long term effect rather than on the dynamic effect, the detailed short-term interaction is irrelevant and is not modeled. Moreover, traffic signals, stop signs and yield signs produce flow that is a proportion of the flow arriving at an intersection. In the long-term point of view, whether traffic signals, stop signs or yield signs are used in a crossing type intersection or not, the flow produced will still be similar, since each cycle loses a small amount of time due to the acceleration and deceleration of vehicles.

The capacity of intersections are designed based on the demand of the traffic flow. If the predicted traffic demand is high, roads are designed to have the greatest capacity possible, subject to certain constraints. In this study, the thinking is reversed. If the capacity of each road link is known, the probability distribution of each link in the intersection can be estimated. The probability distribution can then be turned into a relative traffic flow in steady state condition, which are called *ideal flows*. Traffic flow is modeled as a function of the proportional capacity of *inflow* and *outflow*. Thus, this study aims to answer the following questions:

1) What criteria are necessary so that a crossing type intersection will accommodate the maximum total flow?
2) What is the behavior of the flow in a crossing type intersection with respect to the total inflow capacity and the total outflow capacity?
3) How does the change in the total flow in a crossing type intersection affects network performance indices such as speed, travel time and delay?

The rest of the paper is organized as follows. Section II

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contains a literature review on some articles on how they dealt with intersection capacity analysis. Section III gives the reader an overview of the ideal flow model, so that readers will be familiar with the terms used in this paper. This section contains a discussion of the ideal flow model framework and how it is applied to traffic assignment. Section IV discusses how the model is applied to a crossing type intersection, and the simulation methodology. Results are discussed and conclusions are drawn in sections V and VI respectively.

II. Review of Related Literature

The ideal flow model used both simulation and analytical methods to gain information on the flow capacity of a crossing intersection, aiming also to determine the ideal performance of a road network. Moreover, computing the ideal flow matrix requires the computation of link capacity proportions which are derived from link probabilities.

Similar research on the simulated traffic on unsignalized intersections to get data about its capacity, such as traffic intensity and average waiting time has been done by [6]. Their research compared their simulation results with results from analytical methods based on technical regulations in the Czech Republic. Another research [7] concluded that the performance of pre-timed signalized intersections affects the performance of the whole road network while [8] used “uncertainty analysis” or a probabilistic method in filling in input values when analyzing the capacity of signalized intersections.

III. Overview of the Ideal Flow Model

The ideal flow model is based on a random walk of multi-agents in a directed network graph [9] that is uniformly distributed in a steady state condition. A network is most efficiently utilized when flow is uniformly distributed on it over time and space. When the ideal flow model is based on a uniform probability distribution, the ideal flow network has the characteristic of having *equal outflow* from each node, and that the network entropy is maximized [10]. This model aims to determine the ideal distribution of traffic flow in a network, and this phenomenon happens when the flow is distributed uniformly over time and space.

The probability distribution of the links in a given network graph can be derived from the link probabilities computed as link capacity proportions. If this is the case, the model is still applicable since it can be generalized for any probability distribution using the concept of Markov chains. In a Markov chain, in a very long random walk, the probability that an agent ends at some vertex in a network graph is independent of the agent’s starting point [11]. One of the assumptions, therefore, in this model is that the network graph must necessarily be strongly connected [12]. This implies that the resulting ideal flow matrix is *irreducible*.

The ideal flow network is obtained from the stationary distribution of a simple irreducible stochastic matrix. An absolute ideal flow can be obtained through the scaling property of ideal flow matrices. Since an ideal flow is a relative flow, multiplying an ideal flow matrix $\mathbf{F}$ by a positive scalar κ produces an equivalent ideal flow matrix; that is,

$$\mathbf{F} \equiv \kappa\mathbf{F}, \kappa > 0 \tag{1}$$

The framework for the ideal flow model is as follows:

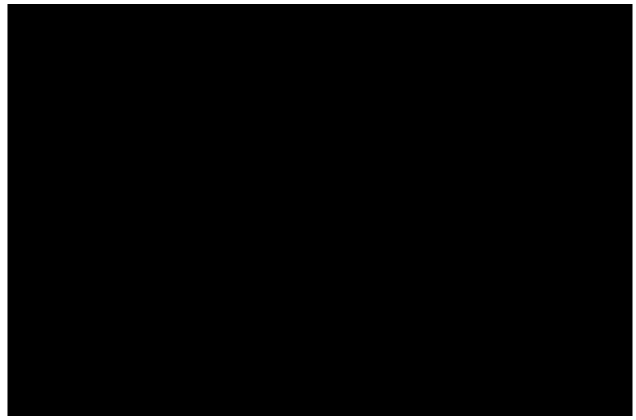

Fig. 1. Ideal Flow Framework for Traffic Assignment in Matrix Form

Suppose a network structure is given with adjacency matrix $\mathbf{A} = [a_{ij}]$. A link distance matrix $\mathbf{L} = [l_{ij}]$, a link capacity matrix $\mathbf{C} = [c_{ij}]$, and a link maximum speed matrix $\mathbf{U} = [u_{ij}]$ can be constructed using the same matrix structure as $\mathbf{A}$. If the matrices are converted such that they will contain only binary elements, the matrices are equal. All the matrices in the framework use the same matrix structure.

The stochastic transition matrix

$$\mathbf{S} = [s_{ij}] = \frac{c_{ij}^{\alpha} e^{\beta c}}{\sum_{j=1}^{O} c_{ij}^{\alpha} e^{\beta c}} \tag{2}$$

is formulated based on the general proportional capacity and is similar in form to the model of generalized cost [13].

It is possible to set the values of the *power sensitivity* parameter α and the *exponential sensitivity* parameter β for each node; however, it is advisable to use a single value for α and a single value for β to be used for the entire network. High values assigned to α and to β tend to make the higher capacity leg of an intersection have higher probability values. When $\alpha = 1$ and $\beta = 0.00001$ (small positive numbers), (2) simplifies to (3). The latter, (3), is called *proportional capacity*.

$$\mathbf{S} = [s_{ij}] = \frac{c_{ij}}{\sum_{j=1}^{O} c_{ij}} \tag{3}$$

The computation of the ideal flow matrix is based on properties of Markov chains, by initially computing for the node probability distribution $\boldsymbol{\pi} = [\pi_i]$. This node probability distribution is obtained by solving

$$(\mathbf{S}^T - \mathbf{I})\boldsymbol{\pi} = 0 \tag{4}$$

subject to the constraints

$$\begin{aligned} \mathbf{j}^T\boldsymbol{\pi} &= 0 \\ \boldsymbol{\pi} &\geq 0 \end{aligned}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{j}^T = \begin{bmatrix} 1 & \dots & 1 \end{bmatrix}$. The proof of the convergence and uniqueness of an irreducible stochastic matrix can be found in [14].

The node probability vector $\boldsymbol{\pi}$ can be solved using singular value decomposition [15]. The solution is obtained using (21). The symbol \ is a MATLAB notation.

$$\boldsymbol{\pi} = \begin{bmatrix} \mathbf{S}^T - \mathbf{I} \\ \mathbf{j}^T \end{bmatrix} \backslash \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \tag{5}$$

The ideal flow matrix $\mathbf{F} = \begin{bmatrix} \mathbf{f}_1^T & \dots & \mathbf{f}_i^T & \dots & \mathbf{f}_n^T \end{bmatrix}^T$ is calculated from

$$\mathbf{f}_i^T = \pi_i \mathbf{s}_i^T \quad \text{for } i = 1, \dots, n \tag{6}$$

Since the stationary distribution $\boldsymbol{\pi}$ is unique [16], the ideal flow matrix, where each row vector is a scalar multiple of the stationary distribution and a row vector of a stochastic transition matrix, is unique for each stochastic transition matrix.

When the ideal flow matrix is obtained, the flow/capacity ratio matrix

$$\mathbf{W} = \begin{bmatrix} w_{ij} \end{bmatrix} = \begin{cases} \frac{f}{c}, & c_{ij} \neq 0 \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

can be derived.

A maximum allowable flow/capacity ratio ξ has to be set for the entire network. The value of ξ depends on the model used when computing for the travel time from the flow/capacity ratio: whether it is based on Greenshields' model [17] or the BPR [18] cost function. In most practical cases, the value should be in the range of $0.9 < \xi < 1.0$.

When the value of ξ is set, the scaling factor κ can be found using

$$\kappa = \frac{\xi}{\max w_{ij}} \tag{8}$$

To anchor the ideal flow matrix from the relative flow into the absolute flow, the scaling property can be applied. The absolute flow matrix $\mathbf{F}$ is computed using the equation

$$f_{ij} = \kappa w_{ij} c_{ij} \quad \text{if } c_{ij} \neq 0 \tag{9}$$

The new ideal flow matrix is the absolute link flow in the network with the same unit as the capacity. For example, if the capacity is in vehicles per hour, then the absolute link flow is also in vehicles per hour.

Since the final flow/capacity ratio $\mathbf{W} = [w_{ij}]$, link distance matrix $\mathbf{L} = [l_{ij}]$, and maximum speed on each link $\mathbf{U} = [u_{ij}]$, the link speed for each link can be computed using Greenshields' [17] model and create a link speed matrix V where

$$\mathbf{V} = [v_{ij}] = \left[\frac{u}{2}(1 + \sqrt{1 - w_{ij}})\right] \tag{10}$$

Clearly, the value of the flow/capacity ratio cannot go beyond one to make the elements of V real numbers.

The travel time matrix, therefore, is

$$\mathbf{T} = [t_{ij}] = \left[\frac{2l}{u\,(1+\sqrt{1-w}\,)}\right] \tag{11}$$

and the minimum travel time, which happens at free flow condition, is

$$\mathbf{T}_0 = [t_{ij}^0] = \left[\frac{l}{u}\right] \tag{12}$$

The delay is the difference between the actual travel time (due to traffic flow) and the minimum travel time. The delay matrix $\mathbf{D}$ is as follows:

$$\mathbf{D} = [\delta_{ij}] = \left[\frac{2l}{u\,(1+\sqrt{1-w}\,)} - \frac{l}{u}\right] \tag{13}$$

The alternate travel time model as derived from BPR [18] is

$$\mathbf{T} = [t_{ij}] = [t_{ij}^0(1 + \gamma(w_{ij})^\eta)] \tag{14}$$

IV. Methodology

A. Intersection Model

The typical crossing intersection network is shown in Fig. 2.

Fig. 2. Core Design of Crossing Intersection

The corresponding link capacity matrix for Fig. 2 is

$$\mathbf{C} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & c_2 & 0 & 0 & 0 \\ c_1 & 0 & c_6 & c_7 & c_8 \\ 0 & c_3 & 0 & 0 & 0 \\ 0 & c_4 & 0 & 0 & 0 \\ 0 & c_5 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Applying the ideal flow model requires that the graph generated must be strongly connected. To ensure this, a dummy cloud vertex and a dummy cloud edge must be introduced to the current network. The dummy edges are extensions of the links they represent. This means that the capacity of each dummy edge will be equal to the capacity of the edge it follows. For instance, the capacity of dummy link za is the same as the that of link ab. Similarly, the capacity of dummy link cz is the same as that of link bc. See Fig. 3.

The corresponding link capacity matrix for Fig. 3, which is irreducible, is

$$
\mathbf{C} = \begin{array}{c|cccccc}
 & a & b & c & d & e & z \\
\hline
a & 0 & c_2 & 0 & 0 & 0 & c_1 \\
b & c_1 & 0 & c_6 & c_7 & c_8 & 0 \\
c & 0 & c_3 & 0 & 0 & 0 & c_6 \\
d & 0 & c_4 & 0 & 0 & 0 & c_7 \\
e & 0 & c_5 & 0 & 0 & 0 & c_8 \\
z & c_2 & 0 & c_3 & c_4 & c_5 & 0
\end{array}
$$

Note that the original network is still the core network. The

Fig. 3. Strongly Connected Design of Crossing Intersection

flow performance indices will be acquired using the values from the core network only, without dummy links nor the cloud node.

Let c_{out} and c_{in} be defined as the total outflow capacity and total inflow capacity respectively, where $c_{out} = c_1+c_6+c_7+c_8$ and $c_{in} = c_2 + c_3 + c_4 + c_5$. Using the configuration of the crossing intersection in Fig. 2, the stochastic matrix $\mathbf{S}$ is

$$
\mathbf{S} = \begin{bmatrix}
0 & \frac{c_2}{c_1+c_2} & 0 & 0 & 0 & \frac{c_1}{c_1+c_2} \\
\frac{c_1}{c} & 0 & \frac{c_6}{c} & \frac{c_7}{c} & \frac{c_8}{c} & 0 \\
0 & \frac{c_3}{c_3+c_6} & 0 & 0 & 0 & \frac{c_6}{c_3+c_6} \\
0 & \frac{c_4}{c_4+c_7} & 0 & 0 & 0 & \frac{c_7}{c_4+c_7} \\
0 & \frac{c_5}{c_5+c_8} & 0 & 0 & 0 & \frac{c_8}{c_5+c_8} \\
\frac{c_2}{c} & 0 & \frac{c_3}{c} & \frac{c_4}{c} & \frac{c_5}{c} & 0
\end{bmatrix}
$$

The node probability distribution vector $\boldsymbol{\pi}$ can be normalized such that one of the entries is 1 (refer to (15)), since its values are relative. In other words, multiplying by a positive scalar produces an equivalent node probability distribution vector that spans the same hyperplane.

$$
\boldsymbol{\pi}^T = \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 & \pi_5 & 1 \end{bmatrix} \tag{15}
$$

The elements of the node probability distribution are listed below. For simplicity, the values are written in terms of π_2.

$$
\begin{aligned}
\pi_1 &= \frac{\frac{c_1}{c} + \frac{c_2}{c}}{1 - \frac{c_1}{c}\frac{c_2-c_1}{c_1+c_2}} \\
&= \pi_2\frac{c_1}{c_{out}} + \frac{c_2}{c_{in}} \\
\pi_2 &= \frac{1 + \frac{c_2-c_1}{c_1+c_2}\frac{c_2}{c}}{1 - \frac{c_2-c_1}{c_1+c_2}\frac{c_1}{c}} \\
&= 1 + \pi_1\frac{c_2-c_1}{c_1+c_2} \\
\pi_3 &= \pi_2\frac{c_6}{c_{out}} + \frac{c_3}{c_{in}} \\
\pi_4 &= \pi_2\frac{c_7}{c_{out}} + \frac{c_4}{c_{in}} \\
\pi_5 &= \pi_2\frac{c_8}{c_{out}} + \frac{c_5}{c_{in}}
\end{aligned}
$$

The ideal flow matrix is

$$
\mathbf{F} = \begin{bmatrix}
0 & \frac{\pi_1 c_2}{c_1+c_2} & 0 & 0 & 0 & \frac{\pi_1 c_1}{c_1+c_2} \\
\frac{\pi_2 c_1}{c} & 0 & \frac{\pi_2 c_6}{c} & \frac{\pi_2 c_7}{c} & \frac{\pi_2 c_8}{c} & 0 \\
0 & \frac{\pi_3 c_3}{c_3+c_6} & 0 & 0 & 0 & \frac{\pi_3 c_6}{c_3+c_6} \\
0 & \frac{\pi_4 c_4}{c_4+c_7} & 0 & 0 & 0 & \frac{\pi_4 c_7}{c_4+c_7} \\
0 & \frac{\pi_5 c_5}{c_5+c_8} & 0 & 0 & 0 & \frac{\pi_5 c_8}{c_5+c_8} \\
\frac{c_2}{c} & 0 & \frac{c_3}{c} & \frac{c_4}{c} & \frac{c_5}{c} & 0
\end{bmatrix}
$$

An interesting property can be observed from ideal flow matrices. An ideal flow matrix is said to be *premagic*. A *premagic matrix* is a square nonnegative matrix where the sum of rows is the same as the sum of columns. In matrix $\mathbf{F}$, the sum of rows is equal to $\boldsymbol{\pi}$ and the sum of columns is equal to $\boldsymbol{\pi}^T$.

The flow/capacity matrix $\mathbf{W}$ can be derived by dividing the nonzero elements of $\mathbf{F}$ by the corresponding elements of $\mathbf{C}$.

$$
\mathbf{W} = \begin{bmatrix}
0 & \frac{\pi_1}{c_1+c_2} & 0 & 0 & 0 & \frac{\pi_1}{c_1+c_2} \\
\frac{\pi_2}{c} & 0 & \frac{\pi_2}{c} & \frac{\pi_2}{c} & \frac{\pi_2}{c} & 0 \\
0 & \frac{\pi_3}{c_3+c_6} & 0 & 0 & 0 & \frac{\pi_3}{c_3+c_6} \\
0 & \frac{\pi_4}{c_4+c_7} & 0 & 0 & 0 & \frac{\pi_4}{c_4+c_7} \\
0 & \frac{\pi_5}{c_5+c_8} & 0 & 0 & 0 & \frac{\pi_5}{c_5+c_8} \\
\frac{1}{c} & 0 & \frac{1}{c} & \frac{1}{c} & \frac{1}{c} & 0
\end{bmatrix}
$$

Note that, similar to $\mathbf{F}$, the elements of $\mathbf{W}$ are relative values. When the elements of $\mathbf{W}$ are multiplied by a positive scalar, an equivalent flow/capacity matrix is obtained.

When the flow/capacity matrix has been obtained, performance indices of the network flow, such as speed, travel time and delay follows from (10), (11), and (13) respectively.

B. Simulation Method for Analysis

The Monte Carlo simulation was implemented on a random sample of 10,000 crossing intersections. The following are the assumptions for the simulation environment:

- The number of lanes is random, ranges from 1 to 10, and is not always an integer, since it is possible to have a fraction of a lane (e.g. half a lane).

- The capacity per lane is set at 2000 vehicles/hour per lane.
- The length of each link is 500 meters.
- The maximum speed is set at 60 km/hour for each link.
- The maximum allowable flow/capacity ratio is set at 0.99.
- Greenshields' cost function is applied.

The capacity ratio index is defined as

$$\lambda = \frac{c_{in}}{c_{out}} \tag{16}$$

This number measures the relationship of the total inflow capacities and total outflow capacities with the total flow.

The values of λ are then plotted against the flow/capacity ratios, the total flow, and some network performance indices such as speed, travel time and delay.

V. Results and Discussion

Figure 4 shows the relationship between the values of λ and the flow/capacity ratios. There is an apparent boundary line in this figure, and it occurs when $\lambda = 1$. This separates the data set into two: (i) when $0 < \lambda < 1$ and (ii) when $\lambda > 1$. Notice the difference in colors of the two sets. These are generated as results of k-means clustering. In the execution of the k-means clustering method, total input lanes and total output lanes are used as features, which correctly defines the grouping seen in Fig. 4.

Fig. 4. Relationship between λ and the flow/capacity ratio

The green curve is a model estimate of the data set. The estimated power regression line is of the form

$$w = \varphi\lambda^{-\theta} \tag{17}$$

The second data set parameters are $\varphi = 0.94324$ and $\theta = -0.35674$.

Fig. 5 shows the relationship between λ and the total flow. Notice that the peak of the total flow occurs when $\lambda = 1$.

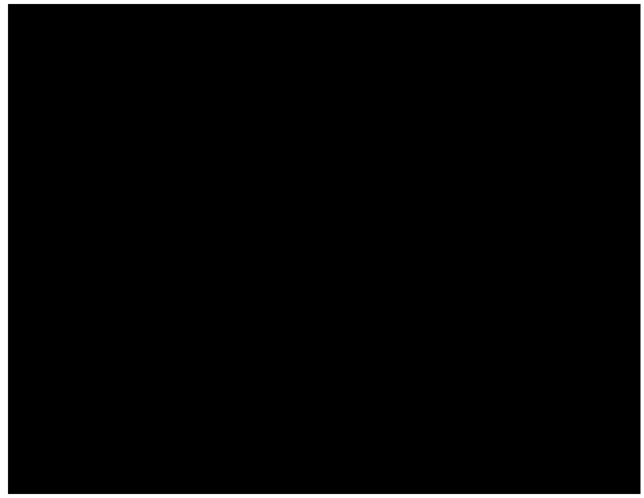

Fig. 5. Relationship between λ and total flow

When the same flow/capacity ratio (equation (17)) is applied to equations (10),(11), and (13), the following equations are obtained:

$$\mathbf{V} = \left[\frac{u}{2}\left(1 + \sqrt{1 - \varphi\lambda^{-\theta}}\right)\right] \tag{18}$$

$$\mathbf{T} = \left[\frac{2l}{u\left(1+\sqrt{1-\varphi\lambda^{-}}\right)}\right] \tag{19}$$

$$\mathbf{D} = \left[\frac{2l}{u\left(1+\sqrt{1-\varphi\lambda^{-}}\right)} - \frac{l}{u}\right] \tag{20}$$

The graphs of the preceding equations are displayed as green lines in Fig. 6, Fig. 7, and Fig. 8 respectively.

Fig. 6. Relationship between λ and average link speed

Observe that each of the graphs in Fig. 4, Fig. 5, Fig. 6, Fig. 7, Fig. 8 is divided into three clusters: (i) when $0 < \lambda < 1$, (ii) when $\lambda > 1$, and (iii) when $\lambda = 1$. In (i), the total inflow capacity is less than the total outflow capacity. In this cluster, an increase in λ also increases the flow (either the total flow

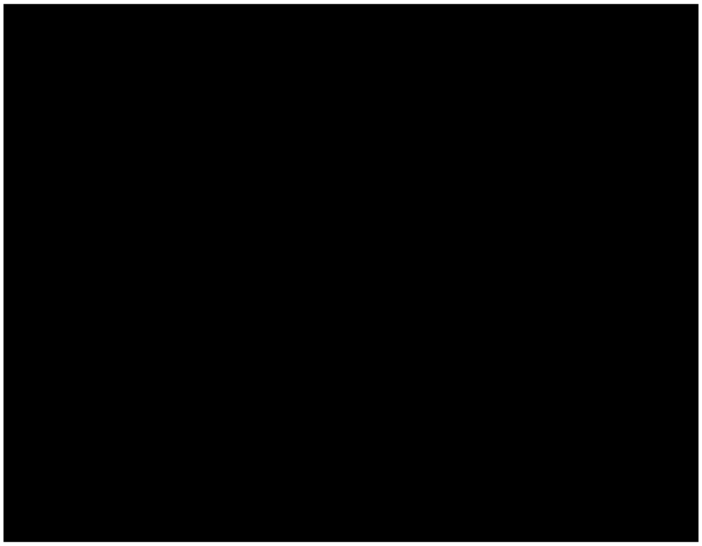

Fig. 7. Relationship between λ and average travel time

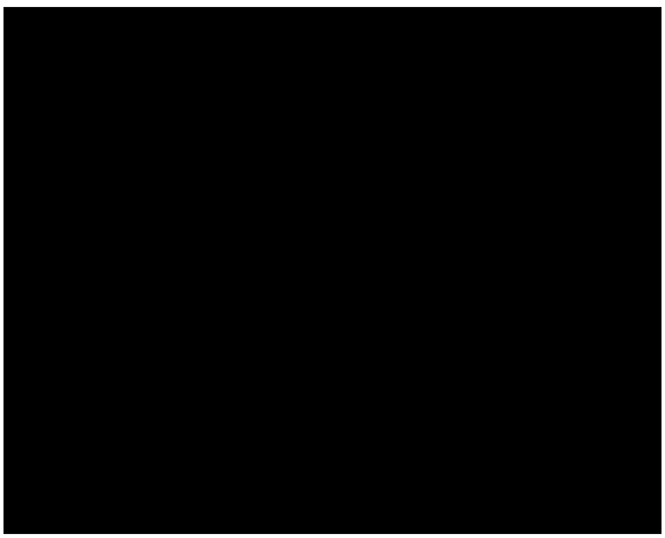

Fig. 8. Relationship between λ and average link delay

or the maximum flow). This means, the greater the value of λ, the better the flow performance.

In (ii), where the total inflow capacity is greater than the total outflow capacity, an increase in λ decreases the flow that can be accommodated by the intersection (even if the number of lanes for each leg is increased). In other words, the greater the value of λ, the worse the flow performance.

In (iii), a special case, where the total inflow capacity is equal to the total outflow capacity, the total flow that can be accommodated by the intersection reaches its maximum value. This is the optimum design of the intersection. In this scenario, the ideal flow matrix becomes equivalent to the capacity matrix. Thus, the flow/capacity ratio reaches the maximum value for each link, which means that the flow performance is maximized. This is shown in the following:

$$\lambda = 1 \rightarrow \mathbf{F} \equiv \kappa \mathbf{C} \rightarrow \mathbf{W} \equiv \xi \text{ if } w_{ij} \neq 0$$

Note that, for all clusters, when the value of the total flow changes, the values of the performance indices are affected. For example, when the total flow increases, the value of speed decreases and the values of travel time and delay increase. Specifically, when the total flow reaches its maximum value, speed will attain its minimum value while travel time and delay will reach their respective maximum values.

VI. Conclusion

The ideal traffic flow can be used as guide to analyze intersection capacity based on steady state condition.

The results of the Monte Carlo simulation produced three clusters of behaviors. These clusters depend on the value of λ, which is defined as the ratio of the total inflow capacity to the total outflow capacity. In the first cluster, where $0 < \lambda < 1$, the behavior of the flow of the crossing type intersection is almost linear. Total flow, travel time and delay increases, and speed decreases, almost linearly when λ increases.

In the second cluster, where $\lambda > 1$, the total flow, travel time, and delay decreases to an asymptotic value when λ increases. Speed, meanwhile, decreases asymptotically to a value, when λ increases.

The third cluster is the special case, when $\lambda = 1$. Based on the ideal flow analysis, the optimum design of crossing intersection capacity happens when the total inflow capacity is equal to the outflow capacity. At that point, the total flow that can be accommodated by an intersection will reach its maximum value.

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VII. Appendix: Numerical Example

Suppose a crossing intersection is given, where the link edge weights correspond to the number of lanes. To make the graph strongly connected, a cloud node and dummy links are added as in Fig. 9. Each lane has a capacity of 2000 vehicles/hour per lane. For simplicity, set the base maximum speed (i.e. the speed at free flow) of each link to 30 km/hour for the first lane. For each increase in the number of lanes, the base maximum speed will be increased by 10 km/hr. Also, let each link have a length of 500 meters. Based on the given network specification,

Fig. 9. Strongly Connected Intersection Design of Numerical Example

the following are the link capacity matrix $\mathbf{C}$, the link maximum speed matrix $\mathbf{U}$ and the link distance matrix $\mathbf{L}$:

$$\mathbf{C} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 1 & 2 & 4 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{U} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 40 & 0 & 0 & 0 \\ 40 & 0 & 30 & 40 & 60 \\ 0 & 50 & 0 & 0 & 0 \\ 0 & 50 & 0 & 0 & 0 \\ 0 & 40 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\mathbf{L} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0.5 & 0.5 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The updated capacity matrix $\mathbf{C}$ is

$$\mathbf{C} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ z \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & z \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 2 \\ 2 & 0 & 1 & 2 & 4 & 0 \\ 0 & 3 & 0 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 & 0 & 4 \\ 2 & 0 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

Use (3) to create the stochastic matrix $\mathbf{S}$. When simulating with a computer, it is preferred to use matrices in the computations. Let C_i denote the sum of the elements of row i in $\mathbf{C}$. Let $\nu = \begin{bmatrix} \frac{1}{C_1} & \frac{1}{C_2} & \cdots & \frac{1}{C} & \cdots & \frac{1}{C} \end{bmatrix}$. Define $\mathbf{G}$ to be a diagonal matrix with diagonal entries equal to the values from ν, i.e., $\mathbf{G} = diag(\nu)$. With $\mathbf{G}$, the stochastic matrix can be obtained by using the equation $\mathbf{S} = \mathbf{GC}$. In this example,

$$\mathbf{S} = \begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{2}{9} & 0 & \frac{1}{9} & \frac{2}{9} & \frac{4}{9} & 0 \\ 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} \\ 0 & \frac{3}{5} & 0 & 0 & 0 & \frac{2}{5} \\ 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ \frac{1}{5} & 0 & \frac{3}{10} & \frac{3}{10} & \frac{1}{5} & 0 \end{bmatrix}$$

The node probability vector $\boldsymbol{\pi}$ can be solved using singular value decomposition [15]. The solution is obtained using (21). The symbol $\backslash$ is a MATLAB notation.

$$\boldsymbol{\pi} = \begin{bmatrix} \mathbf{S}^T - \mathbf{I} \\ \mathbf{j}^T \end{bmatrix} \backslash \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \tag{21}$$

The node probability distribution vector for this example is $\boldsymbol{\pi} = \begin{bmatrix} \frac{626}{1415} & \frac{1033}{947} & \frac{596}{1415} & \frac{307}{566} & \frac{969}{1415} & 1 \end{bmatrix}$. Applying (6), with the greatest common divisor as the scaling factor gives the simple form of the ideal flow matrix. Observe that F is premagic.

$$\mathbf{F} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ z \\ \Sigma \end{array} \begin{array}{cccccc|c} a & b & c & d & e & z & \Sigma \\ 0 & 626 & 0 & 0 & 0 & 626 & 1252 \\ 686 & 0 & 343 & 686 & 1372 & 0 & 3087 \\ 0 & 894 & 0 & 0 & 0 & 298 & 1192 \\ 0 & 921 & 0 & 0 & 0 & 614 & 1535 \\ 0 & 646 & 0 & 0 & 0 & 1292 & 1938 \\ 566 & 0 & 849 & 849 & 566 & 0 & 2830 \\ 1252 & 3087 & 1192 & 1535 & 1938 & 2830 & 11834 \end{array}$$

Set the maximum allowable flow/capacity ratio to $\xi = 0.99$ to get the scaling factor κ. With this, the ideal flow can be bound with the actual capacity in vehicles/hour to obtain the absolute steady flow matrix. The absolute steady flow matrix $\mathbf{F}$ can be obtained using (9). The capacity is converted to flow capacity by multiplying the number of lanes with 2000 vehicles/hour per lane.

$$\mathbf{F} = \begin{array}{c|cccccc|c} & a & b & c & d & e & z & \Sigma \\ \hline a & 0 & 3614 & 0 & 0 & 0 & 3614 & 7727 \\ b & 3960 & 0 & 1980 & 3960 & 7920 & 0 & 17820 \\ c & 0 & 5161 & 0 & 0 & 0 & 1720 & 6881 \\ d & 0 & 5317 & 0 & 0 & 0 & 3544 & 8861 \\ e & 0 & 3729 & 0 & 0 & 0 & 7458 & 11187 \\ z & 3267 & 0 & 4901 & 4901 & 3267 & 0 & 16336 \\ \hline \Sigma & 7727 & 17820 & 6881 & 8861 & 11187 & 16336 & 68313 \end{array}$$

The flow/capacity ratio matrix, based on (7), is

$$\mathbf{W} = \begin{bmatrix} 0 & 0.90 & 0 & 0 & 0 & 0.90 \\ 0.99 & 0 & 0.99 & 0.99 & 0.99 & 0 \\ 0 & 0.86 & 0 & 0 & 0 & 0.86 \\ 0 & 0.89 & 0 & 0 & 0 & 0.89 \\ 0 & 0.93 & 0 & 0 & 0 & 0.93 \\ 0.82 & 0 & 0.82 & 0.82 & 0.82 & 0 \end{bmatrix}$$

Based on the flow/capacity ratio, the average link speed matrix (in km/hour), average link travel time matrix (in minutes), and average link delay matrix (in seconds) can be obtained using (10), (11), and (13) respectively.

$$\mathbf{V} = \begin{bmatrix} 0 & 26.22 & 0 & 0 & 0 & 26.22 \\ 22 & 0 & 16.5 & 22 & 33 & 0 \\ 0 & 34.35 & 0 & 0 & 0 & 20.61 \\ 0 & 33.44 & 0 & 0 & 0 & 26.75 \\ 0 & 25.20 & 0 & 0 & 0 & 37.81 \\ 28.56 & 0 & 35.7 & 35.7 & 28.56 & 0 \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} 0 & 1.14 & 0 & 0 & 0 & 1.14 \\ 1.36 & 0 & 1.82 & 1.36 & 0.91 & 0 \\ 0 & 0.87 & 0 & 0 & 0 & 1.46 \\ 0 & 0.90 & 0 & 0 & 0 & 1.12 \\ 0 & 1.19 & 0 & 0 & 0 & 0.79 \\ 1.05 & 0 & 0.84 & 0.84 & 1.05 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 23.66 & 0 & 0 & 0 & 23.66 \\ 36.82 & 0 & 49.09 & 36.82 & 24.55 & 0 \\ 0 & 16.40 & 0 & 0 & 0 & 27.34 \\ 0 & 17.83 & 0 & 0 & 0 & 22.29 \\ 0 & 26.41 & 0 & 0 & 0 & 17.61 \\ 18.03 & 0 & 14.42 & 14.42 & 18.03 & 0 \end{bmatrix}$$

The average speed is 26.59 km/hour, the average travel time is 1.19 minutes, and the average delay is 28.95 seconds. Note that the flow performance indices are obtained only from the core network, without the dummy links nor the cloud node.