Chapter 11

MESOSCOPIC MULTI-AGENT PEDESTRIAN SIMULATION

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ABSTRACT

Pedestrian model is a superset of mobile traffic agents as it has higher degree of freedom to move than vehicular traffic. This article describes model development of a multi-agent pedestrian simulation at mesoscopic level of detail but at individual level of assemblage. The space is represented as lattice grid similar to Cellular Automata model. However, different from the traditional CA models for pedestrians, several pedestrian agents can occupy a single cell. It consists of only fundamental factors of permission, interaction and navigation. The navigation is guided by sink propagation value using smoothing relaxation algorithm while the interaction is a Beta distribution function of cell density. The simulation is able to illustrate spatial dispersion of pedestrian and queuing behavior of crowd as well as pedestrian way-finding between rooms in a floor plan within static and dynamic environment. It is quite remarkable that this fundamental model could produce route choice self-organization and yield pedestrian behaviors in queue and walking nearer to the shortest path.

Keywords: *mesoscopic, route choice self-organization, sink propagation value, multi-agent, smoothing relaxation.*

INTRODUCTION

Pedestrian model is a superset of mobile traffic agents. Pedestrian movement has a much higher degree of freedom than vehicular movement so it makes vehicular traffic a subset of pedestrian traffic in the simulation. For example, pedestrian is able to turn direction almost

immediately and the environment that pedestrians move is 3 dimensional space (e.g. using elevator and escalator). Therefore, vehicular movement can be viewed as a special case of pedestrian movement model given its many constraints.

Pedestrian simulation is an abstract representation of reality into mathematical equation or computer program that might give a new paradigm to evaluate the outcome of various design, control and policy scenarios upon pedestrian related facilities. As everyone is a pedestrian at some point of his/her journey, pedestrian facilities are not merely crosswalk and sidewalk but also comprise of public facilities and open space, airport, train and bus station, public park, car parking, inside supermarket or mall, hospital, stadium and so on including alley inside office, ship and airplane. Basically it is very easy to change layout and rules of those facilities within the simulation and it is very costly to have such change in reality. That makes pedestrian simulation a great tool as 'pedestrian laboratory' to experiment with many types of facilities and behavioral rules and what-if scenarios.

In the past decade, there are many pedestrian model have been developed. This occurrence is not surprising considering the significant amount of applications range from evacuation, business, and crowd control as well as urban planning. These models can be categorized in general into two approaches (see Figure 1). First approach is to consider the long term and steady state effect of the system by modeling pedestrian flow as continuous entity similar to fluid dynamic (e.g. Henderson, 1974). The reality that pedestrians are actually discrete dynamic entities generates better second approach of discrete pedestrian dynamic. Among discrete pedestrian dynamic simulation, we can distinguish two level of abstraction. The first is level of interaction among pedestrians and between pedestrian with the facilities, and the second is level of assemblage.
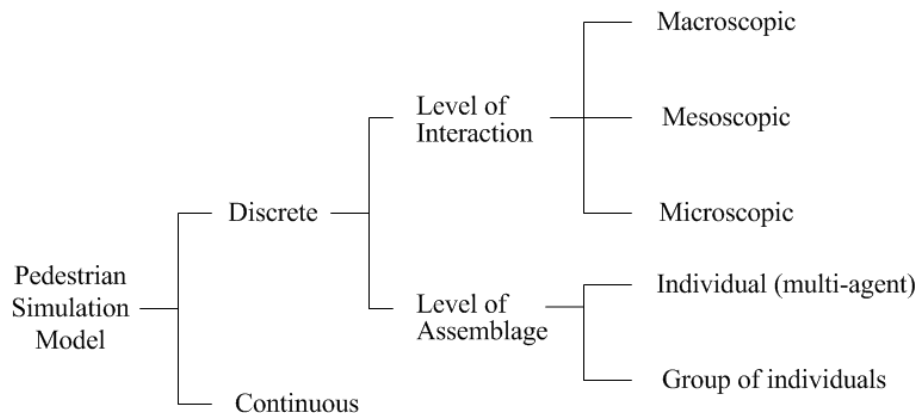

Figure 1. Category of pedestrian simulation model.

Most of recent pedestrian simulations tend to model at *microscopic* level of interaction where an individual pedestrian are modeled as an entity that occupy a space or a single cell (Blue and Adler (2000), Bierlaire et al (2003), Gloor et al (2004), Teknomo and Gerilla (2005a), Kretz and Schreckenberg 2006)). In such microscopic model, much of the attention of the modeler is on the detail interaction between pedestrians and between pedestrian with the environment through a set of parameters and a set of models that produce agent's intention to move with certain velocity or acceleration. For detail design and relatively short period of simulation time, microscopic pedestrian models produce excellent results. The

problem with microscopic simulation, however, is the computational speed limitation. To simulate a large number of pedestrians over wide area and long time periods, higher level of detail is needed.

In this chapter, we describe slightly a higher level of interaction which is called *mesoscopic* level. The focus is not on single pedestrian interaction but on more aggregation of several pedestrians in a region (e.g. room, hall, railway platform etc.). Different from other previous pedestrian mesoscopic simulations (e.g. Hanisch et al (2003)) that model only the flow of pedestrian, the model described in this article is agent-based in which each agent will keep their own timing to enter and go out of a cell. In contrast to microscopic model that model the interaction between agents in high detail such as imposing collision detection behind the model, our mesoscopic model put the pedestrian interaction between agents as aggregation model of speed-density functions over space.

Modeling pedestrians as multi-agents at mesoscopic level of interaction has several advantages compare to both region-based mesoscopic model and microscopic cellular automata model. In most practical applications, modeling pedestrian in a very high level of detail with collision detection has very limited usefulness only to deal with specific behavior and visualization. In the analysis, however, such microscopic model will always be aggregated into either mesoscopic level or macroscopic level. This make us to think that if we don't really need those level of detail in the result of the analysis, why don't we start at mesoscopic level? The mesoscopic level is much simple to model and much faster to compute with a proper program). We can model it in such a way without reducing much on the spatial level of detail, similar to the result of microscopic simulation. Another reason of using mesoscopic level of interaction is because in most cases, things that make pedestrian microscopic simulation computationally expensive are the detail interaction of collision detection and collision avoidance which are not considered necessary.

Recent pedestrian models at mesoscopic level are found in Hanisch et al (2003) and Tolujew and Alcalá (2004) which discussed simple region-based mesoscopic pedestrian simulation for train station. The environment is divided into several storage regions. Pedestrian movement is deterministic and highly directed by the connection of regions based on hypothetical distribution function of moving time. Pedestrians are not represented individually but only as counted number of pedestrian in the storage over time. In general, these kinds of mesoscopic simulations tend to model pedestrian flow as fluid dynamic flow rather than as discrete entity. The models assumed the existing of large number of pedestrians and long period of simulation time (steady state condition).

Coarser *macroscopic* level of interaction in which pedestrians are further aggregated as pedestrian flow move on a graph could reduce the computational speed significantly but with much lost of information. To show spatial detail such as when and where formation of queue and observation on arc-formation of queue in front of a door, mesoscopic level of interaction is essential.

In contrast to the argument of Florian et al (2001) and Hanisch et al (2003) that mesoscopic level means groups of pedestrians as an entity that move together, we model a single pedestrian as an individual agent. In this case, we distinguish level of interaction (micro-meso-macro) with level of assemblage. Aggregating group of pedestrian as single entity yield much lost of information about individual trajectory from which many flow performances can be derived.

Mesoscopic level of detail however is also have its own limitation over the microscopic one, in which the detail visualization of pedestrian movement in term of detail spatial occupation (e.g. will a pedestrian turn to right or left) and detail pedestrian behavior (e.g. how they avoid collision) are omitted.

Pedestrian Mesoscopic Model

The mesoscopic model explain in this section is working in a region with obstruction. The space is represented as a regular lattice with discrete cells to cover the regions. The cell is a square where the side of the cell represents possible movement direction. Unlike microscopic cellular automata (CA) model that model an individual pedestrian to occupy a single cell, our cell size is slightly larger to cover several pedestrians on a single cell. In this sense, our model is coarser than microscopic CA. A region is defined as regular lattice in which the pedestrian movements are not uniform for the whole region but only uniform in a cell. In this sense, our model has finer spatial detail where the speeds and densities at all cells are computed at each simulation time step. A region is also a bounded space in which each pedestrian will only have a single origin and a single destination. Thus definition of a region is not defined by wall or facility but by how the pedestrian will move. A region is divided into square[1] cells say between 1 by 1 meters up to 3 by 3 meters. The size of the cell must be small enough compare to the area of the region to be able to show some spatial detail, and at the same time the area of the cell must be large enough that the time for a pedestrian to go *in* and the time to go *out* are not an instant. When the cell size is 0.5 by 0.5 and each cell is maximum density is one, we get microscopic cellular automata model similar to Kretz and Schreckenberg (2006) or Schadschneider (2001). In this sense, the mesoscopic model is more general model than the cellular automata. There is a short travel time which depends on the cell diameter and pedestrian speed inside the cell. Cell diameter s is an average of inner circle and outer circle diameters of a lattice cell. The inner and outer circles are shown in Figure 2.
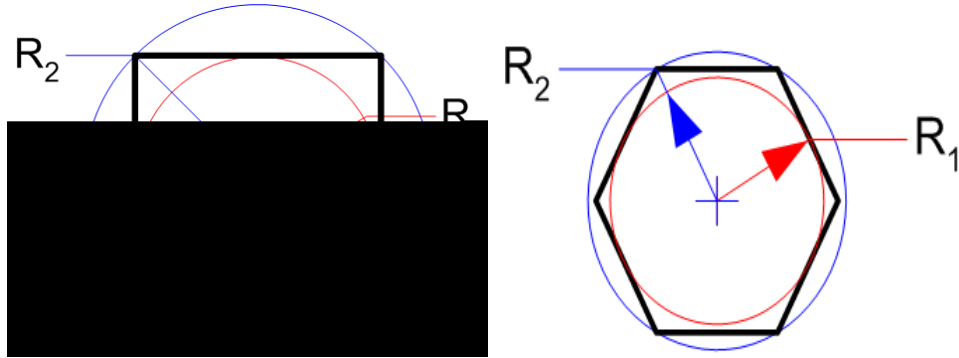

Figure 2. Lattice cell diameter is an average of inner circle and outer circle.

The mesoscopic pedestrian simulation model can be described as follow. We have matrix $\mathbf{D}$ represents the count of pedestrians on each cell and matrix $\mathbf{L}$ indicate the layout of the facility. Zero value represents obstruction or wall while value of one indicates free space or

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1 Alternatively, it can be hexagonal cells but we don't use it in this article.

permission for pedestrian to move. Sink cells has high value represent attraction strength to navigate. Let $\mathbf{x}_t$ denotes the current position of a pedestrian agent at discrete time stamp t. A *neighborhood function* $\mathbf{N_M} = f\left(\mathbf{M}, \mathbf{x}_t^i\right)$ takes current agent position and a matrix $\mathbf{M}$ as input and produces 3 by 3 Moore neighborhoods matrix which values are taken from matrix $\mathbf{M}$ with $\mathbf{x}_t$ as the center position. Let *Neighborhood probability* $\mathbf{N}_p$ is a normalized entry-wise product of neighborhood values of layout permission $\mathbf{N_L}$, probability to enter the next cell $\mathbf{N_C}$ and heading probability $\mathbf{N_Q}$, as shown in the equation below.

$$\mathbf{N}_p = \left[\!\left[\mathbf{N_D} \bullet \mathbf{N_L} \bullet \mathbf{N_Q} \right]\!\right] \tag{1}$$

Then, movement vector of each agent is defined as

$$\mathbf{v}_t = \arg\max \mathbf{N}_p \tag{2}$$

where $\mathbf{N_D}$, $\mathbf{N_L}$ and $\mathbf{N_Q}$ are obtained by inputting respectively matrix $\mathbf{D}$, $\mathbf{L}$ and $\mathbf{Q}$ as matrix $\mathbf{M}$ in the neighborhood function .

Layout permission $\mathbf{N_L}$ represents subset of layout matrix where the pedestrian agent currently located. The layout matrix $\mathbf{L}$ may contain of four types of cells: free space, source, sink and obstruction cells. Obviously, obstruction cell is not permissible to be entered by any agents, free space has maximum density limitation while source and sink do not have any density limitation. Probability to enter the next cell $\mathbf{N_D}$ is a function of speed-density relationship and the probability is determined based on the current count of pedestrians in the neighborhood of the agent. Since the number of pedestrians in a cell is discrete, this function can be simplified as a function to lookup table based on current density. Matrix $\mathbf{Q}$ represents heading probability that will be explained later in Pedestrian Navigation section. Operator $\bullet$ is Hadamard product or entry-wise product of the two matrices, while symbol $\left[\!\left[\mathbf{M} \right]\!\right]$ indicate normalization function by dividing each entry of the matrix $\mathbf{M}$ with its maximum value. The next agent position is simply summation of current position with the movement vector

$$\mathbf{x}_{t+dt} = \mathbf{x}_t + \mathbf{v}_t \tag{3}$$

As mentioned earlier, each agent is treated as individual pedestrian and not as aggregation of several pedestrians. Each agent has its own position (in cell coordinate), speed, and recorded time to enter and out of a cell and record the trajectory path for flow performance determination.

An agent will move out of a cell only when $t \geq t_{in} + {s}/{u}$. Symbol u is the average speed of the current cell. Let ξ denote the maximum number of pedestrians that can be

accommodate in a single cell, and $c(\mathbf{x})$ represent the number of pedestrians in cell $\mathbf{x}$. Pedestrian agent will reconsider the movement vector to the next possible location (including staying in the current position) when $c\left(\mathbf{x}_{t+dt}^{i}\right) = \xi$.
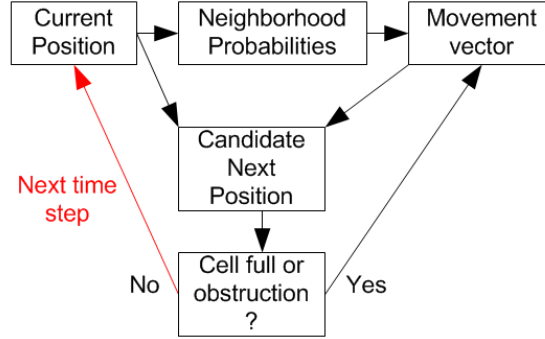

Figure 3. Structure of agent's movement.

The structure of the simulation step of each agent is shown in the figure 3. Neighborhood probabilities are determined based on the current position and movement vector is computed based on neighborhood probability. Then the next position is computed based on movement vector and current position. Additional examination whether the cell is full (the density is higher than maximum density of that cell) or the next position is an obstruction is needed to ensure that the agent will behave correctly. This examination is analogue to the collision detection in microscopic multi agent simulation but much simpler. This step is done in parallel *after* the entire pedestrian agents evaluate their candidates of next positions. Thus we make a buffer matrix represent temporary count of next position and use this temporary count to decide the next movement vector (the second or third possible next location and so on including current position are considered in the order of neighborhood probabilities). When this examination is passed, in the next time step, the next position is considered as the current position.

The mesoscopic model has only two sets of internal parameters. The first parameter is positive scalar value represents strength of attraction located on the sink cells. Low strength of attraction does not give enough "strength" to allow pedestrian to overcome obstruction. This is due to the nature of smoothing relaxation algorithm that does not distinguish enough heading probability among neighboring cells if the strength of attraction values is relatively small. Relatively small strength of attraction causes pedestrians to move around some regions without ability to reach destination. In general, however, setting high strength of attraction (i.e. > 100) yields the same movement pattern regardless the strength of attraction value. Some critical value of strength of attraction exists for certain layout to move the pedestrian from cyclical movement pattern.

The second set parameter is speed-density function related to probability to enter a cell. The second parameter is located in each cell in the lattice. Different type of facilities such as stair, elevator and escalator would have different set of speed-density function. If u denotes the speed in meter/second and k is the density (represented by a discrete number of

pedestrians per cell area), the theoretical relationship between speed and density can be generalized by setting it as an approximation function of a cumulative Beta distribution.

$$u = 1 - \frac{B_k(\alpha, \beta)}{B(\alpha, \beta)}$$

(4)

The nominator $B_x(\alpha, \beta)$ is the Incomplete Beta Function formulated as

$$B_x(\alpha, \beta) = \int_0^x g^{\alpha-1}(1-g)^{\beta-1} dg$$

(5)

The denominator $B(\alpha, \beta)$ is the Beta function formulated similar to the Incomplete Beta function with $x = 1$, which is

$$B(\alpha, \beta) = \int_0^1 g^{\alpha-1}(1-g)^{\beta-1} dg$$

(6)

The range of two parameters is $\alpha > 0$ and $\beta > 0$. The linear function is a special case when $\alpha = \beta = 1$.

Pedestrian Navigation

In absent of obstruction such as wall, a room with a table in the middle, etc. pedestrian agents can move forward and avoid obstruction but cannot find a way to the destination if the destination is behind the obstruction. Only based on layout permission $\mathbf{N_L}$, probability to enter the next cell $\mathbf{N_D}$, pedestrian agent will be able to move from source cells to sink cells if and only if the line connecting origin and destination (OD line) do not pass any obstruction. To make pedestrian able to navigate from one place to another in the present of obstruction, we introduce heading probability matrix $\mathbf{Q}$.

Heading probability is simply a normalized index $[0, 1]$ of each cell that drives the pedestrian agent. The real value of heading probability value is not important. However, the comparison heading probability values of one cell to the other within neighborhood is important to guide pedestrian movement. Lowest heading probability is connected to source cells and highest heading probability is in the sink cells. The value of heading probability of a cell is depending on the distance and direction of the cell from the destination (i.e. sink cell). Nearer the cell to destination and more direct way to reach destination, lead to higher heading probability. Putting heading probabilities of all cells into a single matrix, we obtain

navigation matrix. Implicitly, the basic assumption on this navigation matrix is perfect information on the routes.

Navigation matrix is computed based on indices that we called *Sink Propagation Value* (SPV). Sink propagation value (SPV) is a scalar whose value is depend on the neighborhood, the value is monotonically increasing (or decreasing) by the distance of the cell from sink basin. The concept of sink propagation value is a generalization based on our observation that the results of several methods such as Reinforcement-Learning (i.e. Q-Learning), Smoothing Relaxation, Bellman Flooding algorithm and Distance Transform applied on regular lattice grid or network graph produce some common properties that the values of the lattices cell are propagating from sink cells into all other connected cells. Knowing that interconnection between adjacent lattice cells can be generalized into network graph, we define those common properties as sink propagation value. More formally, Sink Propagation Value (SPV) is a vertex value that holds the following properties:

1) Positive: $v_i \geq 0$

2) Zero at sink basin: $v_s = 0$

3) Infinity if sink is unreachable: $v_i = \infty \Leftrightarrow i \nrightarrow s$

4) Monotonic increasing by the shortest distance from sink: $v_i \geq v_j \Leftrightarrow d_{is} \geq d_{js}$, where d_{is} is the shortest distance from node i to node sink s

There are many ways to compute navigation matrix with above-mentioned properties for heading probability. We have tried to use dual of Q learning (Teknomo, 2005) with successful result but rather slow to compute. We presented here faster way to obtain heading probability matrix through *relaxation* method (Winston, 1993). Heading probability described in this article can be viewed as a scalar function in contrast to vector function of potential field described by Hoogendoorn, and Bovy (2004).

Relaxation method arbitrates between smoothness expectations and the actual data based on local constraints. It is smoothing or numerical interpolation method to fill the gap between measurements based on confidence level of the data. Suppose we have a measurement matrix **B** and each data has a corresponding confidence level **C** ($0 \leq c_i \leq 1$), we want to interpolate our measurement data. The interpolated data is denoted by matrix **A**. The smoothing by relaxation method is performed using the following steps:

1) At first, interpolate the value

$$a^0(i, j) = b(i, j) \cdot c(i, j) + \left(1 - c(i, j)\right) \cdot \frac{b(i-1, j) + b(i+1, j) + b(i, j-1) + b(i, j+1)}{4}$$

Set $b(i, j) = c(i, j) = 0$ if there is no measurement in that location.

2) After that, do the following iteration until the difference values of $a(i, j)$ between two consecutive iterations is small.

$$a^{k+1}(i,j) = a^k(i,j) \cdot c(i,j) + \left(1 - c(i,j)\right) \cdot \frac{a^k(i-1,j) + a^k(i+1,j) + a^k(i,j-1) + a^k(i,j+1)}{4}$$

Heading probability matrix or also called Navigation matrix $\mathbf{Q}$ is obtained by setting layout matrix $\mathbf{L}$ as the measurement matrix and a binary matrix $\mathbf{C}$ as the confidence level. Binary matrix $\mathbf{C}$ is obtained from layout matrix such that sinks and obstruction cells have unit confidence level while free space has zero confidence level indicate there is no measurement only in the free space cells. If $\mathbf{A} = \text{Relaxation}(\mathbf{B}, \mathbf{C})$ represents smoothing function by relaxation method, then heading probability matrix is computed as $\mathbf{Q} = \text{Relaxation}(\mathbf{L}, (\mathbf{L} < 1 \| \mathbf{L} > 1))$. Navigation matrix can be pre-computed for each origin and destination pair before the simulation to save the computation time.

Simulation Results

The mesoscopic multi-agent model was implemented for simulation of five scenarios. The first two scenarios are simple cases of one room and two rooms with single door to illustrate how the simulations work. The last three scenarios are extension of the first two scenarios to demonstrate self-organization phenomena in route choice. In all scenarios, the size of the layout is about 20 by 30 meters and attraction strength parameter is set as high value (i.e. 200) to ensure continuity and stability pedestrian movements with linear speed density relationship at maximum 5 pedestrians per square meter at zero speed and 1.4 meter/second free flow speed. The probability to enter a cell also has linear relationship to the density with zero probability at maximum density and 1 at free flow speed. Source and sink cells are 5 meter width.

The first scenario is the simplest case where a number of pedestrians move from one door on the left to the other door on the right in a room without any obstruction. The second scenario is a simple demonstration of navigation where the pedestrian agents move from one room to the other passing through as narrow door. In both scenarios, the total number of pedestrians is 50 and time is measured in second.

The layout matrices $\mathbf{L}$ of the first and second scenario are shown in left side of Figure 4. Source or origin door and sink or destination door are annotated in the figures. For the first scenario, source and sink cells are located vertically in the middle of the layout. For the second scenario, the source cells are located horizontally at top left of the layout while the sink cells are located vertical at top right. Door width is 3 meter. Heading probability matrices $\mathbf{Q}$ of the first two scenarios are shown in right side of Figure 4. Brighter color near the sink indicates higher probability values while lower probabilities correspond to darker color. Notice the contour pattern around the sink indicate equal probability values.

Figure 5 exhibits some image sequences of the simulation of the first and second scenarios in term of pedestrian density. These two scenarios shows that pedestrians tend to flock together by walking nearer to the shortest path although it impose some cost of more walking delay due to pedestrian interaction rather than spreading themselves to the other part of the empty regions. This phenomenon is plausible as often observed in similarity to

pedestrian movement pattern in a crossing where the pedestrians do not spread over the space but flock together along the zebra cross. Spatial queuing pattern can also be observed near the door of destination following the arc pattern of **Q** matrix. Simulation with higher pedestrian flow produces similar pattern and but slower to move and produce higher delay. Scenario 2 illustrates pedestrian navigation based on the relaxation method

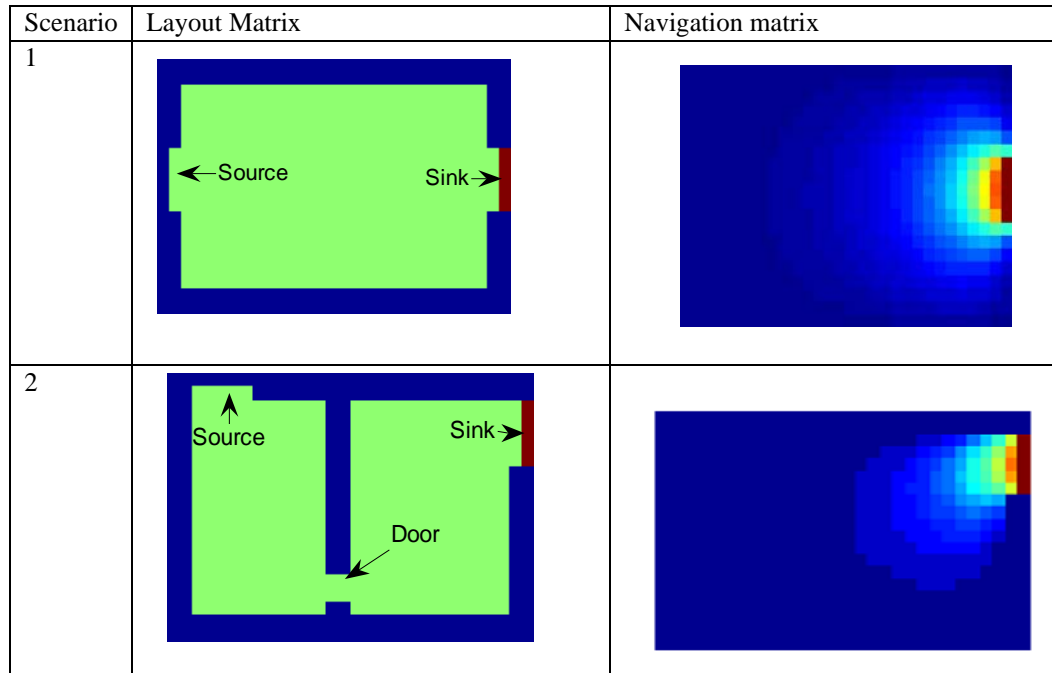| Scenario	Layout Matrix	Navigation matrix
1		
2		

Figure 4. Layout matrix (left) and heading probability matrix (right) of scenario-1 of one room without obstruction and scenario-2 of two rooms with a single door.

Self-Organization in Route Choice

The next three scenarios are designed on single layout of two rooms connected by two doors of equal 3 meter width. The purpose is to demonstrate how route choice happens as self-organization phenomena. The three scenarios differ in term of number of pedestrians generated in the source cells. Pedestrians are generated at once in the source cells with equal distribution among source cells. Figure 6 shows the layout matrix and heading probability matrix used for these three scenarios. The source is on the top left and the sink is on the bottom right. As the pedestrian agents move from source to sink in this layout, there are only two alternative routes. The first route is through North door which produce shorter route than the second route though South door.

The image sequences of the three scenarios are exhibited in Figure 7. Scenario-3 has 20 pedestrians and all of them move through North door because the door width is enough to accommodate them. Scenario-4 has 40 pedestrians and as the queue build up in the North door, there are a few of the pedestrian agents (3 pedestrians) manage to use South door.

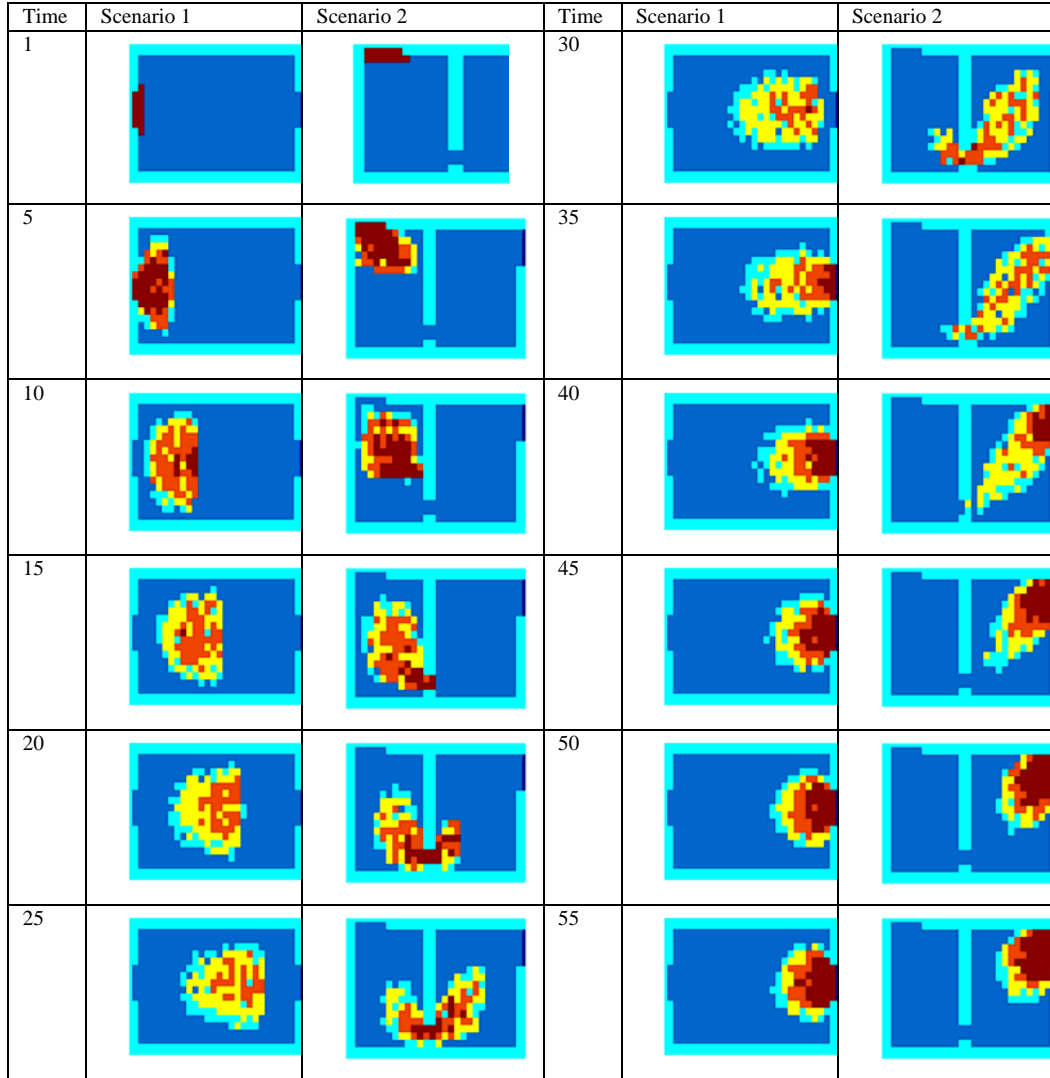| Time	Scenario 1	Scenario 2	Time	Scenario 1	Scenario 2
1			30		
5			35		
10			40		
15			45		
20			50		
25			55		

Figure 5. Movement pattern of pedestrian flow in two scenarios of one room without obstruction and two rooms with a single door.

Scenario-5 has 80 pedestrians and as the queue in the North door is higher than the scenario-4, the pedestrian are spread spatially to the south and some of these agents (about 10 to 20 agents) are moving through South door.

As explained in the previous sections, the proposed mesoscopic multi agent model do not have explicit model for route choice. In fact, the route choice happen as output of the model as we increase the number of pedestrians flow. The choice of route occurs as self-organization phenomena to find optimum ways to go to destination. This phenomenon is an emergence behavior in which the arrangement arises from the interaction of agents rather than as the result of centralized rule in the model. When the number of pedestrian flow is relatively small there is no queue to pass the door, the pedestrian agents will use the shortest path. At certain level of pedestrian flow, a few pedestrian agents goes to the other part of the regions

and found better ways to get to destination. As the number of pedestrian flow increases, there are more pedestrians move to the south and found another entry.
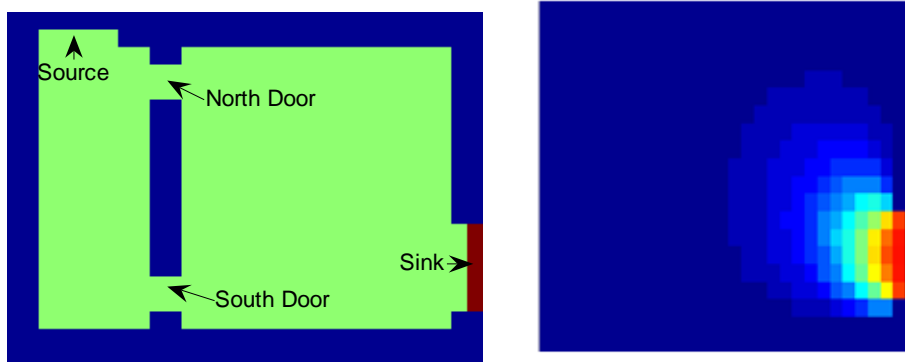

Figure 6. Layout matrix (left) and navigation matrix (right) of scenario number 3-5, two rooms is connected by two doors of equal width.

The choice of route take place due to spatial dispersions and the spatial dispersion take place because of the queue cause by the increasing number of pedestrian flow. Internally the principle of route choice self-organization happens due to balance between interaction (which is the local information) and navigation (global information). When we narrow the North door width into 1 meter, the queue build up is faster and at critical number of pedestrian of 15, there is single pedestrian move through the South door. Shorter distance between doors, the higher probability pedestrian will pass through the South door.

Conclusions and Further Studies

We have modeled pedestrian at mesoscopic level of detail using multi agent paradigm without imposing expensive collision detection and collision avoidance. We maintain such model to be mesoscopic in the level of interaction and still individual multi-agent based in the level of assemblage. The interaction between pedestrian is represented by speed-density function over space. The simple pedestrian movement model is only a function of spatial layout, lookup function of speed-density table over current density and navigation matrix. The navigation matrix is based on sink propagation value with smoothing relaxation algorithm of spatial lattice permission. It is quite appealing that this minimal model could produces self-organization in route choice and yield several pedestrian behaviors in queue and walking nearer to the shortest path.

It was found that choice of route occurs as emergence behavior to find optimum ways to go to destination due to spatial queuing dispersions. High pedestrian flow and distance between alternative routes and door width was found to be main factors of creating such self-organization phenomena.

There are more questions raised than answer provided in this introductory article. There are many challenges ahead to calibrate such model spatially and to compare the flow performance of more interesting scenarios than simple scenarios provided in this article. The

effect of changing parameters such as strength of attraction and speed density relationship in other type of facilities are also subject of further studies.

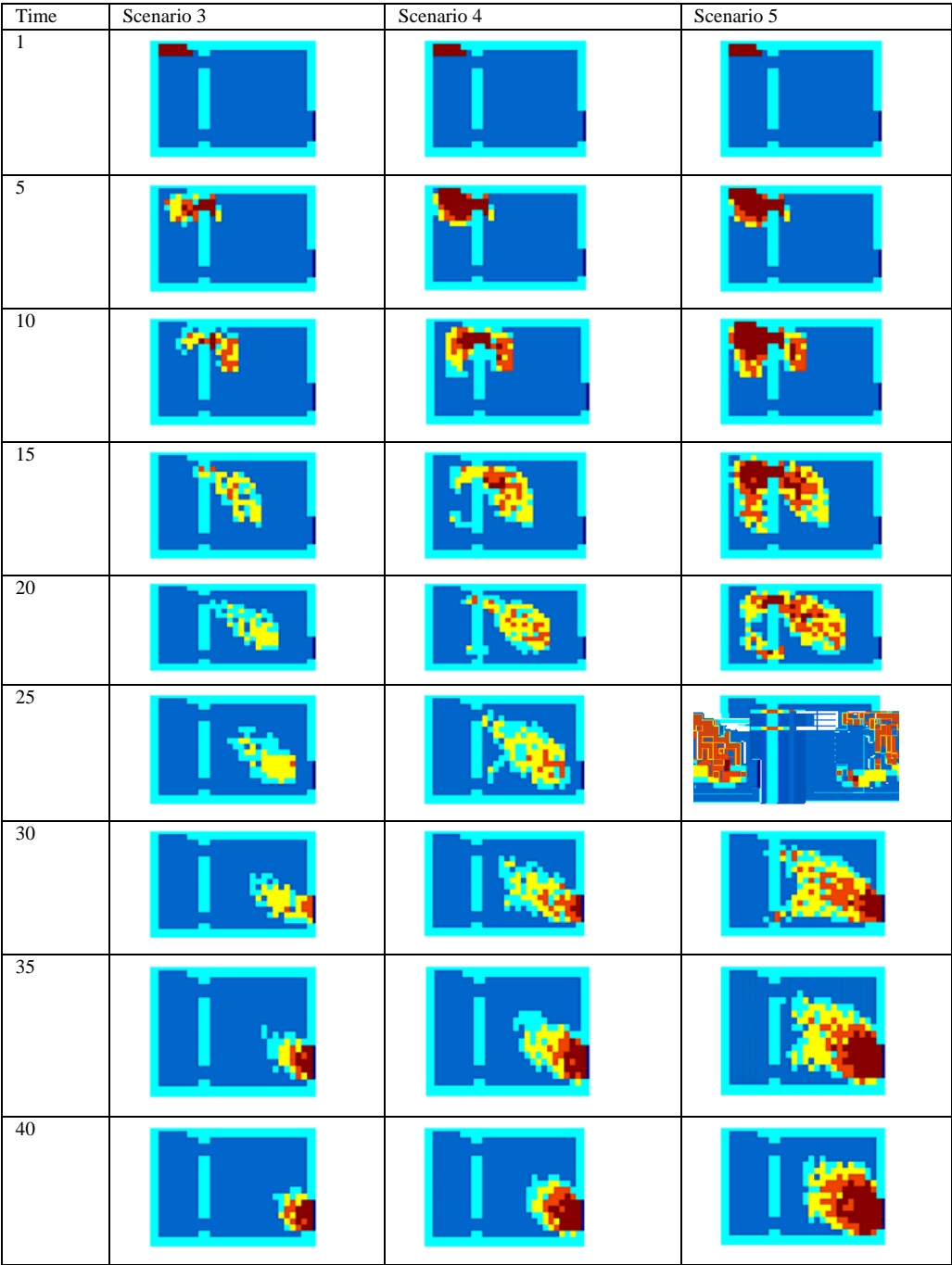| Time	Scenario 3	Scenario 4	Scenario 5
1			
5			
10			
15			
20			
25			
30			
35			
40			

Figure 7. Movement pattern of pedestrian indicates self-organized route choice happen as a function of pedestrian flow. Scenario 3, 4 and 5 respectively have 20, 40 and 80 pedestrians.

ACKNOWLEDGEMENT

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